

STAT 134 - Instructor: Adam Lucas

Practice Midterm 2 problems

Monday, November 16, 2020

Print your name: _____

SID Number: _____

Exam Information and Instructions:

- You will have 52 minutes to take this exam. Closed book/notes/etc. No calculator or computer.
- We will be using Gradescope to grade this exam. Write any work you want graded on the front of each page, in the space below each question. Additionally, write your SID number in the top right corner on every page.
- Provide calculations or brief reasoning in every answer.
- Unless stated otherwise, you may leave answers as unsimplified numerical and algebraic expressions, and in terms of the Normal c.d.f. Φ . Finite sums are fine, but simplify any infinite sums.

GOOD LUCK!

1. Beloved Stat 134 students Jonny and Ethan have very different study habits for the final. Jonny quits studying at a constant hazard rate $\lambda = \frac{1}{2}$ (i.e. average 2 hours). Ethan randomly quits anytime between 0 and 4 hours (also average 2 hours). You may assume Jonny and Ethan study independently. What is the chance the first quitter stops studying between 2 and 3 hours.
2. For each distribution of X indicated in the parts below, evaluate $Var(X)$ as a simple fraction, without involving the constants c_1, c_2, c_3 .

(a) X with values in $[0, 1]$ and density $f_X(x) = c_1 x^2 (1 - x)^3$ for $0 < x < 1$.

(b) X with values in $[0, \infty)$ and density $c_2 x^4 e^{-3x}$ for $0 \leq x < \infty$

3. The joint density of X and Y is

$$f(x, y) = \begin{cases} \frac{4y}{x} & \text{for } 0 < x < 1 \text{ and } 0 < y < x \\ 0 & \text{otherwise} \end{cases}$$

Find the following:

- (a) $E(XY)$
 - (b) The marginal density of X
4. Let U follow a Uniform $(0, 1)$ distribution. Find the distribution of $-\log(U)$.
 5. 10 friends plan to arrive to lunch at noon. The arrival time of each friend is uniform from 11:45 am to 12:15 pm and independent of everyone else.
 - (a) How many minutes after 11:45 is the 3rd earliest person expected to arrive?
 - (b) What is the variance of the arrival time of the 3rd earliest person?
 - (c) Find chance that 3 friends arrive before noon, 5 friends arrive between noon and 12:10, and the last 2 friends arrive between 12:10 and 12:15.
 - (d) Find the chance that all 10 friends arrive within 10 minutes of one another.
 6. $X \sim \mathcal{N}(0, 1)$ the standard normal distribution.
 - (a) What is $\mathbb{E}X^3$?
 - (b) What is $\mathbb{E}X^4$?

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Scratch Paper

Discrete

name and range	$P(k) = P(X = k)$ for $k \in \text{range}$	mean	variance
uniform on $\{a, a + 1, \dots, b\}$	$\frac{1}{b - a + 1}$	$\frac{a + b}{2}$	$\frac{(b - a + 1)^2 - 1}{12}$
Bernoulli (p) on $\{0, 1\}$	$P(1) = p; P(0) = 1 - p$	p	$p(1 - p)$
binomial (n, p) on $\{0, 1, \dots, n\}$	$\binom{n}{k} p^k (1 - p)^{n - k}$	np	$np(1 - p)$
Poisson (μ) on $\{0, 1, 2, \dots\}$	$\frac{e^{-\mu} \mu^k}{k!}$	μ	μ
hypergeometric (n, N, G) on $\{0, \dots, n\}$	$\frac{\binom{G}{k} \binom{N - G}{n - k}}{\binom{N}{n}}$	$\frac{nG}{N}$	$n \left(\frac{G}{N}\right) \left(\frac{N - G}{N}\right) \left(\frac{N - n}{N - 1}\right)$
geometric (p) on $\{1, 2, 3, \dots\}$	$(1 - p)^{k - 1} p$	$\frac{1}{p}$	$\frac{1 - p}{p^2}$
geometric (p) on $\{0, 1, 2, \dots\}$	$(1 - p)^k p$	$\frac{1 - p}{p}$	$\frac{1 - p}{p^2}$
negative binomial (r, p) on $\{0, 1, 2, \dots\}$	$\binom{k + r - 1}{r - 1} p^r (1 - p)^k$	$\frac{r(1 - p)}{p}$	$\frac{r(1 - p)}{p^2}$

Continuous

† undefined.

name	range	density $f(x)$ for $x \in$ range	c.d.f. $F(x)$ for $x \in$ range	Mean	Variance
uniform (a, b)	(a, b)	$\frac{1}{b-a}$	$\frac{x-a}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
normal $(0, 1)$	$(-\infty, \infty)$	$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$	$\Phi(x)$	0	1
normal (μ, σ^2)	$(-\infty, \infty)$	$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(x-\mu)^2/\sigma^2}$	$\Phi\left(\frac{x-\mu}{\sigma}\right)$	μ	σ^2
exponential (λ) = gamma $(1, \lambda)$	$(0, \infty)$	$\lambda e^{-\lambda x}$	$1 - e^{-\lambda x}$	$1/\lambda$	$1/\lambda^2$
gamma (r, λ)	$(0, \infty)$	$\Gamma(r)^{-1} \lambda^r x^{r-1} e^{-\lambda x}$	$1 - e^{-\lambda x} \sum_{k=0}^{r-1} \frac{(\lambda x)^k}{k!}$ for integer r	r/λ	r/λ^2
chi-square (n) =gamma $(\frac{n}{2}, \frac{1}{2})$	$(0, \infty)$	$\Gamma(\frac{n}{2})^{-1} (\frac{1}{2})^{\frac{n}{2}} x^{\frac{n}{2}-1} e^{-\frac{x}{2}}$	as above for $\lambda = \frac{1}{2}$. $r = \frac{n}{2}$ if n is even	n	$2n$
Rayleigh	$(0, \infty)$	$x e^{-\frac{1}{2}x^2}$	$1 - e^{-\frac{1}{2}x^2}$	$\sqrt{\frac{\pi}{2}}$	$\frac{4-\pi}{2}$
beta (r, s)	$(0, 1)$	$\frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)} x^{r-1} (1-x)^{s-1}$	see Exercise 4.6.5 for integer r and s	$\frac{r}{r+s}$	$\frac{rs}{(r+s)^2(r+s+1)}$
Cauchy	$(-\infty, \infty)$	$\frac{1}{\pi(1+x^2)}$	$\frac{1}{2} + \frac{1}{\pi} \arctan(x)$	†	†
arcsine =beta $(1/2, 1/2)$	$(0, 1)$	$\frac{1}{\pi\sqrt{x(1-x)}}$	$\frac{2}{\pi} \arcsin(\sqrt{x})$	$\frac{1}{2}$	$\frac{1}{8}$