# *STAT 134: Midterm 2 Review Sheet*

*Adam Lucas*

*November 16, 2020*

This review sheet is meant to provide a high level overview of concepts, techniques, and methods within the scope of the second midterm.

## *Some common aspects of densities*

a. *Constant and variable parts of densities:* Let $X$ have density $f_X(x) = cx^3e^{-6x}$, where $c$ is a constant. What is the distribution of $X$? Use this to evaluate $\int_0^\infty 5x^3e^{-6x}dx$.

b. *Independence and dependence:* How do we show that two random variables with densities are independent or dependent?

c. *Working with densities:* Remind yourself of the general approach for each of the following situations:

   (i) Obtaining $F_X$ from $f_X$, and vice versa;

   (ii) Obtaining $f_X(x)$ from $f_{X,Y}(x,y)$;

## *Relationships Between Distributions*

We have seen frequently how recognizing the transformations of random variables into other known distributions greatly simplifies problems. In the context of continuous RV problems, the variable at hand is often found by one of these transformations. For each of the following below, identify the distribution that results.

a. $cX$, where $X \sim \text{Exp}(\lambda)$, $c > 0$. We have proved this result using three different methods; recall these arguments.

b. Consider $X = U_{(3)}$ and $Y = U_{(7)}$, the 3rd and 7th order statistics of 10 iid Unif $(0,1)$ RVs. What is the distribution of $X/Y$?

c. $X^2 + Y^2$, where $X, Y$ are i.i.d. standard Normal. What is $\sqrt{X^2+Y^2}$?

## *Symmetry*

a. Under some conditions, we can quickly recognize the expectation of a random variable $X$ to be zero. What are they?

b. Let $X, Y$ be independent $\mathcal{N}(0,\sigma^2)$ random variables. Without using $\Phi$, find $P(X+2Y > 0, X > 0)$. What is the reason behind this answer?

## *Simplifying an infinite sum*

Many distributions and problems in this class involve possible values up to $+\infty$. As we do not accept infinite sums, it is worth reviewing techniques for simplifying these problems.

Let $Y$ have density $f_Y(y) = \frac{\lambda}{2}e^{-\lambda|y|}$, for $y \in \mathbb{R}$. With little computation, find $\mathbb{E}(|Y|)$ and $\mathbb{E}(Y)$.

## *Could You Rephrase That?*

Often we can simplify probability expressions that may be difficult to directly compute by rephrasing them in terms of equivalent events, or using complements.

a. Let $X \sim$ Gamma $(r, \lambda)$, where $r$ is a positive integer. Use what we know about the Poisson process to obtain the CDF of $X$.

b. Let $X$ be an arbitrary random variable with an invertible CDF $F_X$. What is the random variable formed by $F_X(X)$?

## *Some Useful Results*

The following are some results we have previously demonstrated in class which you may take for granted on the final (with some statement/justification of what you are using), but may not be on the reference sheet. Work to prove these results on your own if you can; if you are stuck, the proofs should be in your lecture notes.

a. Let $X \sim$ Exp $(\lambda_X)$, $Y \sim$ Exp $(\lambda_Y)$. What is $P(X < Y)$?

b. Find the distribution of $\min\{X_1, X_2, \ldots, X_n\}$, where the $X_i$'s are independent Exp $(\lambda)$ variables.

c. Suppose cars and trucks arrive at a bridge according to independent Poisson processes with rates $\lambda_c$ and $\lambda_r$ per minute respectively. Given that $n$ vehicles arrive in $t$ minutes, what is the distribution of $N_{c,t}$, the number of cars to arrive by time $t$?