Stat 134 Fall 2019: Midterm Conceptual Review

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This review sheet contains a selection of conceptual and computational problems designed to help you prepare for the midterm. It is important to note however that this sheet is far from comprehensive; you should use this sheet sparingly alongside other resources (such as reviewing lecture notes and recommended problems) to prepare for the midterm. Happy studying and best of luck next week!

"What Does Each Distribution Do?"

For each of the following distributions, identify the type of random variable for which this distribution is an appropriate model.

1. Binomial(n, p): *Counting the number of successes in n independent trials, where the probability of success in each trial is p.*
2. Hypergeometric (n, N, G):
3. Geometric (p) on $\{1, 2, 3, \ldots\}$:
4. Geometric (p) on $\{0, 1, 2, \ldots\}$:
5. Negative Binomial (r, p) on $\{r, r+1, r+2, \ldots\}$:

"When Can I Use This Approximation?"

In this course, we have discussed two main techniques to approximate the Binomial(n, p) distribution. Identify the conditions under which each of the two approximations below can be used. Make sure you also identify what the parameters of the approximated distribution will be in terms of n and p.

1. Normal (μ, σ^2):
2. Poisson (μ):

It is also possible to use the Binomial to approximate the Hypergeometric distribution. Under what conditions can we do so? Identify what the parameters of the Binomial will be in terms of the parameters of the Hypergeometric. Intuitively, why is this approximation appropriate?

"What Distribution Should I Use?"

For each random variable given below, identify which of the distributions you know will be a good model and briefly explain why. Make sure you also identify the parameters (and possible values, if necessary) of the distribution.

It is a good idea to find the possible values for each random variable and then check that the possible values match that of your proposed distribution.

1. The number of sixes in 15 rolls of a fair six-sided die: *$Bin(15, 1/6)$. Fixed number of trials; trials are independent; probability of success is constant; the possible values for this event are $\{0, 1, \ldots, 15\}$ as in the Binomial.*

The level of detail in this example may seem to be overkill but it's a good idea to be as detailed as this in your answers to ensure that you truly understand why each distribution is a good fit.

2. The number of games of Roulette I must play until I win three times, if I only ever bet on black:

The probability of winning in Roulette by betting on black is always 18/38.

3. The number of Aces in 5 cards drawn from a standard deck:
4. Balls are drawn with replacement from a box with 5 red balls and 45 non-red balls until a red ball is drawn. The number of balls drawn from this box:
5. Balls are drawn with replacement from a box with 20 blue balls and 30 non-blue balls until a blue ball is drawn. The number of non-blue balls drawn from this box:

Why is this problem any different from the previous one?

6. Customers arrive at a checkout counter randomly, at a rate of 10 per hour. The number of customers that arrive at the checkout counter in 30 minutes:
7. A fair 20-sided die is rolled 50 times. The face of the die shows a number greater than 15 in 30 of the rolls. The number of sixes in the 50 rolls:

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"Do You Have Any Recommended Problems?"

1. A jar contains r red marbles and b blue marbles. A marble is chosen at random and its color noted. The marble is then replaced, along with m more marbles of the same color (so that there are now $r + b + m$ balls in the jar).

   What diagram could you draw here that might prove helpful?

   i. Find the probability that the second marble is red.
   ii. Given that the second marble is red, what is the probability that the first marble is blue?

2. Roll a 20-sided die 10 times. Find the probability of getting:

   Pay close attention to counting factors in these problems.

   i. At least one six
   ii. At least one five and one six
   iii. 3 sixes given that no number larger than 12 appears
   iv. 3 sixes, 2 twelves, 4 twos, and 1 eight
   v. 1 triple, 3 doubles, and 1 single

   A triple is three rolls of the same kind (similar for double and single).

3. A fair die is tossed 1200 times. Find, using an approximation:

   i. The probability of getting more that 400 sixes
   ii. A number m such that the probability of getting between $200 - m$ and $200 + m$ sixes is approximately 95%.

4. Roll a fair n-sided die until you roll a one, and record this number of rolls. Now, repeat this experiment 10 times, recording the number of rolls each time. Let X = minimum number of rolls in the 10 trials and Y = maximum number of rolls in the 10 trials. For $k > 0$, find:

   i. $P(X > k)$
   ii. $P(Y > k)$

5. Let X have the Poisson(μ) distribution. Find:

   i. $\mathbb{E}[X(X-1)]$
   ii. $\mathbb{E}[X^2]$
   iii. $\mathbb{E}[X(X+1)]$

6. Draw cards from a standard deck until three Aces have appeared. Let X = number of cards drawn. Find:

   i. $P(X > x)$
   ii. $P(X = x)$
   iii. $\mathbb{E}X$ as a simple fraction
   iv. Var(X) using the method of indicators

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