

Stat 134 Fall 2019: Practice Midterm Questions

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THIS REVIEW SHEET contains a selection of practice problems designed to help you prepare for the midterm. The topics range through all material that we've seen before, but be aware these problems are far from comprehensive.

As you work through the problems, ask yourself the following questions: What techniques/methods are you using? If you are using a named distribution, why are you choosing that distribution? Can you relate these problems to some problems we've seen before? What are the differences? Reflect on your thought process, and proceed carefully through each problem.

Practice Midterm Questions

1. Flip a coin 8 times. Given that there are 3 heads, find the probability that:
 - a. The heads are not consecutive.
 - b. The heads are all next to each other.
 - c. There are 2 heads next to each other, and 1 other head by itself.

2. Suppose a coin lands heads with probability p . Toss the coin until you observe three heads. Given your third head occurred on toss n , what is the chance your first head landed on toss k , for $k \in \{1, 2, \dots, n-2\}$?

3. In a room of n people, we are interested in finding out how many people share the same birthday. Assume as before that birthdays are independent and uniformly distributed across the year. Let N represent the number of people who share a birthday with at least one other person.
 - a. Calculate $\mathbb{E}(N)$;
 - b. (Warning: Long) Calculate $\text{Var}(N)$.

4. Let X be a Geometric (p) random variable on $\{1, 2, 3, \dots\}$.
- Evaluate $\mathbb{E}(X^{-1})$. Hint: recall the Taylor series, $\log(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$.
 - For p close to 1, what is this value, approximately? Your answer should be a very simple expression in terms of p .
5. Suppose there are two kinds of lottery tickets A and B. You buy A once a day and B once a week. You have a $1/25$ chance of winning A if you buy it, and a $1/4$ chance of winning B. You really want to win, so you keep buying both of them for a year (that is, 365 A tickets and 52 B tickets in total). Let X be the number of winning tickets you have in total. Find:
- $\mathbb{E}(X)$.
 - $\text{Var}(X)$.
 - Approximately $P(X = 18)$.