Berkeley Stat 134, Spring 2016 **Prof. Adhikari**

Final Exam

1. I have three coins. Two of them are fair and the other lands heads with chance 0.9. I pick one of the three coins at random and toss it 10 times.

a) Given that I picked the unfair coin, what is the chance that I get 8 heads?

b) Given that I got 8 heads, what is the chance that I picked the unfair coin?

2. Let X and Y have joint density given by

$$\begin{aligned} f(x,y) &= 2(x+y), \quad 0 \le x \le y \le 1 \\ &= 0 \qquad\qquad \text{otherwise} \end{aligned}$$

a) Find the density of X.

b) Find $E(X)$.

3. Suppose that $X_1, X_2, \ldots$ are lengths of telephone calls, independent and identically distributed with expectation 6 minutes and SD 5 minutes. For each probability below, provide an exact value or an approximation or a bound, whichever is the best answer based on the given information.

a) $P(T > 60 \text{ minutes})$ where $T = \sum_{i=1}^{4} X_i$

b) $P(M > 6.5 \text{ minutes})$ where $M = \frac{1}{400}\sum_{i=1}^{400} X_i$

4. Suppose that each time I place a bet, I win with probability p independently of all other bets. Let l and w be two positive integers such that $l < w$. Suppose I decide to place bets until I have won w bets or lost l bets, whichever happens first. Let X be the number of bets that I place.

a) What are the possible values of X?

b) Find the distribution of X.

c) Given that $X = w$, what is the chance that I have won w bets?

5. Travelers arrive at an airport Information desk according to a Poisson process at the rate of 15 per hour. Assume that each traveler arriving at the desk has a 60% chance of being male and a 40% chance of being female, independent of all other travelers.

a) Fill in the blank with a number: The fifth male traveler is expected to arrive at the desk _________ minutes after the first male traveler.

b) Find the chance that the fifth male traveler arrives at the desk more than 30 minutes after the first male traveler.

c) Find the expected number of female travelers who arrive at the desk before the fifth male traveler.

6. A drawer contains s black socks and s white socks, where s is a positive integer. I pull two socks out at random without replacement and call that my first pair. Then I pull two socks out at random without replacement from the remaining socks in the drawer, and call that my second pair. I proceed in this way till I have s pairs and the drawer is empty.

Let D be the number of pairs in which the two socks are of different colors.

a) Find $E(D)$.

b) Find $Var(D)$.

7. Let M be a student's score on the midterm of a class and F the student's score on the final of the same class. Suppose that M and F have a bivariate normal distribution with correlation 0.6. Assume that:

- $E(M) = 70, \ \ SD(M) = 8$
- $E(F) = 65, \ \ SD(F) = 10$

a) Find the chance that the student scores above average on both the midterm and the final.

b) Find the chance that the student scores higher on the final than on the midterm.

8. Let X and Y be independent random variables such that X has the exponential distribution with rate α and Y has the exponential distribution with rate β.

a) Let $r > 0$ be a constant. Use the joint density of X and Y to derive a formula for $P(Y > rX)$ in terms of α, β, and r.

b) What are the possible values of the random variable $X/(X + Y)$? Define the cumulative distribution function (c.d.f.) of $X/(X+Y)$. You do not need to provide a formula for it in this part.

c) Use parts **a** and **b** above to find the c.d.f. of $X/(X+Y)$. In the case $\alpha = \beta$, recognize this as the c.d.f. of one of the famous distributions and provide its name and parameters.

9. Candidate A and Candidate B are contesting an election. There are n voters, each of whom votes for exactly one of the two candidates.

Assume that each voter votes for Candidate A with chance p and for Candidate B with chance $1 - p$, independently of all other voters.

You can assume that n is large and that neither candidate is an overwhelming favorite: the constant p is somewhere between 0.4 and 0.6.

Let X_A be the proportion of voters who vote for Candidate A and X_B the proportion who vote for Candidate B. Let $M = X_A - X_B$ be the "margin of victory" for Candidate A. Note that M can be negative.

Sketch the probability histogram of M, and **justify your choice of shape.** Find $E(M)$ and $SD(M)$ in terms of p and n, and mark them appropriately on your sketch.

10. Let Z_i, $1 \le i \le 4$ be independent standard normal variables. Let $V = \sqrt{Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2}$.

a) Find the density of V.

b) For $v > 0$, find $P(V > v)$.

11. Let X have the exponential distribution with rate 1.

a) Find $P(2.5 < X < 3.5)$.

b) Let A be the event "the integer closest to X is odd". Find $P(A)$.

c) Let B be the event "X is greater than 4". Show that events A and B are independent.

12. Let N have the Poisson distribution with mean μ. Let $U_1, U_2, \ldots$ be independent uniform $(0, 1)$ variables, independent of N.

Let $M = \min(U_1, U_2, \ldots, U_N)$. If $N = 0$, define M to be 1.

a) Find $E(M|N)$.

b) Find $E(M)$.

c) Find the survival function of M.

d) Sketch the c.d.f. of M.

End of Berkeley Spring 2016 Stat 134 Final, A. Adhikari. Please note that exam instructions did not allow answers to be left as integrals (except in terms of Φ) or as infinite sums.