Stat 134: Section 2

Adam Lucas

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Problem 1

Events A, B, and C are defined in an outcome space. Find expressions for the following probabilities in terms of $P(A)$, $P(B)$, $P(C)$, $P(AB)$, $P(AC)$, $P(BC)$, and $P(ABC)$.

a. The probability that exactly two of A, B, C occur.

b. The probability that exactly one of these events occurs.

c. The probability that none of these events occur.

For these questions, it might prove helpful to first draw a Venn diagram.

In part c, use what you already know to avoid doing unnecessary work.

Ex 1.3.10 in Pitman's Probability

Problem 2: Boole's Inequality

The inclusion-exclusion formula gives the probability of a union of events in terms of probabilities of intersections of the various subcollections of these events. Because this expression is rather complicated, and probabilities of intersections may be unknown or hard to compute, it is useful to know that there are simple bounds. Use induction on n to derive Boole's inequality:

Recall the four basic steps in induction: base case, assumption, induction, and conclusion.

$$P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$$

Ex 1.3.13 in Pitman's Probability

Problem 3

A hat contains n coins, f of which are fair, b of which are biased to land heads with probability $2/3$. A coin is drawn from the hat and tossed twice. The first time it lands heads, and the second time it lands tails. Given this information, what is the probability that it is a fair coin?
Ex 1.R.11 in Pitman's Probability

Again, it might prove helpful to first draw a diagram here; use a tree diagram this time.

Problem 4

An experimenter observes an event A as the outcome of a particular experiment. There are three different hypotheses, H_1, H_2, and H_3, which the experimenter regards as the only possible explanations of event A. Under hypothesis H_1, the experiment should produce result A about 10% of the time, under H_2 about 1% of the time, and under H_3 about 39% of the time. Having observed A, the experimenter decides that H_3 is the most likely explanation, and that the probability that H_3 is true is

$$39\%/(10\% + 1\% + 39\%) = 78\%$$

a. What assumption is the experimenter implicitly making?

b. Does the probability 78% admit a long-run frequency interpretation?

c. Suppose the experiment is a laboratory test on a blood sample from an individual chosen at random from a particular population. The hypothesis H_i is that the individual's blood is of some particular type i. Over the whole population it is known that 50% of individuals have blood type 1, 45% have blood type 2, and the remaining proportion have type 3. Revise the experimenter's calculation of the probability of H_3 given A, so that it admits a long-run frequency interpretation. Is H_3 still the most likely hypothesis given A?

Ex 1.5.6. in Pitman's Probability