Stat 134: Section 11

Adam Lucas

March 4th, 2019

Conceptual Review

a. Suppose X, Y are independent Poisson RVs with means μ, λ respectively. What is the distribution of $X + Y$?

b. What are the assumptions needed for a Poisson Process/Scatter?

Problem 1

Suppose X, Y, and Z are independent Poisson random variables, with parameters μ_X, μ_Y, μ_Z respectively. Find:

a. $P(X + Y = 4)$

b. $\mathbb{E}((X + Y + Z)^2)$

Hint: Recall the equation $Var(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$.

From Ex 3.5.11 in Pitman's Probability

Problem 2: Poisson Thinning

Cars arrive at a toll booth according to a Poisson process with rate μ per minute. Suppose each car has a chance p of being an import, and chance $q = 1 - p$ of being domestic, independently of all other cars. Let X_t, Y_t be the number of imports and domestics, respectively, arriving in t minutes. Show that X_t and Y_t are independent Poisson processes. Hint: Start with

$$P(X = x, Y = y) = P(X + Y = x + y)P(X = x \mid X + Y = x + y)$$

Problem 3: Properties of the Geometric Distribution

Let $X_1, X_2 \overset{\text{i.i.d.}}{\sim} \text{Geom}\,(p)$ on $\{0, 1, 2, \ldots\}$.

a. *Memoryless property:* Show that for all $k, m \geq 0$, $P(X_1 = m + k \mid X_1 \geq k) = P(X_1 = m)$. Provide an explanation for why this must be the case, in terms of sequences of successes and failures.

b. *Sums of geometrics:* Let $Y = X_1 + X_2$. What is the distribution of Y? Find $P(X_1 = k \mid Y = n)$, for $0 \leq k \leq n$. (What distribution does this remind you of?)

Prepared by Brian Thorsen and Yiming Shi