# *Stat 134: Section 13*

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## *Conceptual Review*

Consider a Poisson Process with rate $\lambda$ per unit time. Identify what each random variable represents, and find the distributions of:

a. $N_t$;

b. $W_k$;

c. $T_k$. (How is this different from (b)?)

## *Problem 1*

Suppose calls are arriving at a telephone exchange at an average rate of one per second, according to a Poisson arrival process. Find:

a. the probability that the fourth call after time $t = 0$ arrives within 2 seconds of the third call;

b. the probability that the fourth call arrives by time $t = 5$ seconds;

c. the expected time at which the fourth call arrives.

*Ex 4.2.5 in Pitman's Probability*

## *Problem 2: Gammas, Exponentials, and Moments*

Consider the gamma function $\Gamma(r) = \int_0^\infty x^{r-1}e^{-x}dx, \quad r > 0.$

a. Use integration by parts to show that $\Gamma(r+1) = r\Gamma(r)$.

b. Deduce from (a) that for any positive integer $n$, $\Gamma(n) = (n-1)!$

c. Show that if $T \sim$ Exp (1), then $\mathbb{E}(T^n) = n!$.

d. Show that if $S = T/\lambda$, then $S \sim$ Exp ($\lambda$). (Note: from this, we can easily show that $\mathbb{E}(S^n) = n!/\lambda^n$ ).

Hint: Consider the expression $P(S > s)$, then substitute for $S$ appropriately.

*Ex 4.2.9 in Pitman's Probability*

## *Problem 3*

Shocks occur to a system according to a Poisson process of rate $\lambda$. Suppose that the system survives each shock with probability $\alpha$, independently of other shocks, so that its probability of surviving $k$ shocks is $\alpha^k$. What is the probability that the system is surviving at time $t$?

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