Stat 134: Section 14

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Conceptual Review

Consider a Poisson Process with rate λ per unit time. Identify what each random variable represents, and find the distributions of:

a. N_t;

b. W_k;

c. T_k. (How is this different from (b)?)

Problem 1

Suppose calls are arriving at a telephone exchange at an average rate of one per second, according to a Poisson arrival process. Find:

a. the probability that the fourth call after time $t = 0$ arrives within 2 seconds of the third call;

b. the probability that the fourth call arrives by time $t = 5$ seconds;

c. the expected time at which the fourth call arrives.

Ex 4.2.5 in Pitman's Probability

Problem 2: Geometric from Exponential

Show that if $T \sim \text{Exp}(\lambda)$, then $Z = \text{int}(T) = \lfloor T \rfloor$, the greatest integer less than or equal to T, has a geometric (p) distribution on $\{0, 1, 2, \ldots\}$. Find p in terms of λ.

How can we use the CDF of Z to simplify this problem?

Ex 4.2.10 in Pitman's Probability

Problem 3: Gammas, Exponentials, and Moments

Consider the gamma function $\Gamma(r) = \int_0^\infty x^{r-1} e^{-x} dx, \quad r > 0$.

a. Use integration by parts to show that $\Gamma(r+1) = r\Gamma(r)$.

b. Deduce from (a) that for any positive integer n, $\Gamma(n) = (n-1)!$

c. Show that if $T \sim \text{Exp}(1)$, then $\mathbb{E}(T^n) = n!$.

d. Show that if $S = T/\lambda$, then $S \sim \text{Exp}(\lambda)$. (Note: from this, we can easily show that $\mathbb{E}(S^n) = n!/\lambda^n$).

Hint: Consider the expression $P(S > s)$, then substitute for S appropriately.

Ex 4.2.9 in Pitman's Probability