

## *Stat 134: Section 14*

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### *Problem 1*

Suppose calls are arriving at a telephone exchange at an average rate of one per second, according to a Poisson arrival process. Find:

- a. the probability that the fourth call after time  $t = 0$  arrives within 2 seconds of the third call;
- b. the probability that the fourth call arrives by time  $t = 5$  seconds;
- c. the expected time at which the fourth call arrives.

*Ex 4.2.5 in Pitman's Probability*

### *Problem 2*

Local calls are coming into a telephone exchange according to a Poisson process with rate  $\lambda_{loc}$  calls per minute. Independently of this, long-distance calls are coming in at a rate of  $\lambda_{dis}$  calls per minute. Write down expressions for probabilities of the following events:

- a. exactly 5 local calls and 3 long-distance calls come in a given minute;
- b. exactly 50 calls (counting both local and long distance) come in a given three- minute period;
- c. starting from a fixed time, the first ten calls to arrive are local.

*Ex 4.rev.13 in Pitman's Probability*

*Problem 3: Gammas, Exponentials, and Moments*

Consider the gamma function  $\Gamma(r) = \int_0^\infty x^{r-1}e^{-x}dx$ ,  $r > 0$ .

- Use integration by parts to show that  $\Gamma(r + 1) = r\Gamma(r)$ .
- Deduce from (a) that for any positive integer  $n$ ,  $\Gamma(n) = (n - 1)!$
- Show that if  $T \sim \text{Exp}(1)$ , then  $\mathbb{E}(T^n) = n!$ .
- Show that if  $S = T/\lambda$ , then  $S \sim \text{Exp}(\lambda)$ . (Note: from this, we can easily show that  $\mathbb{E}(S^n) = n!/\lambda^n$ ).

Hint: Consider the expression  $P(S > s)$ , then substitute for  $S$  appropriately.

*Ex 4.2.9 in Pitman's Probability*