Stat 134: Section 15

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Conceptual Review

What are the different functions we have used to characterize (i.e., fully describe) distributions of random variables? We have seen four.

Problem 1

Suppose we have a random variable X with continuous and strictly increasing CDF F_X. Find the distribution of $F_X(X)$.

Problem 2: Geometric from Exponential

Show that if $T \sim \text{Exp}(\lambda)$, then $Z = \text{int}(T) = \lfloor T \rfloor$, the greatest integer less than or equal to T, has a geometric (p) distribution on $\{0, 1, 2, \ldots\}$. Find p in terms of λ.
Ex 4.2.10 in Pitman's Probability

How can we use the CDF of Z to simplify this problem?

Problem 3

Let $U_{(1)}, \ldots, U_{(n)}$ be the values of n i.i.d. Uniform (0,1) variables arranged in increasing order. For $0 < x < y < 1$, find simple formulae for:

a. $P(U_{(1)} > x, U_{(n)} < y)$

b. $P(U_{(1)} > x, U_{(n)} > y)$

c. $P(U_{(1)} < x, U_{(n)} < y)$

d. $P(U_{(1)} < x, U_{(n)} > y)$

Ex 4.6.3 in Pitman's Probability

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