Stat 134: Section 19

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Conceptual Review

a. Let X, Y have joint density $f_{X,Y}(x, y) > 0$ for all $x, y > 0$. Set up an integral that would yield the density of $Z = X + Y$.

b. Repeat (a), but for $W = 3X + 4Y$.

c. For $X \sim$ Exp (λ), $a > 0$, what is the distribution of aX?

Problem 1

Let $X \sim$ Unif (0,1), and $Y \sim$ Unif (0,2), independent of each other.
Find the density of $Z = X + Y$, using:

a. the convolution formula;

b. the CDF of Z.

Problem 2: Competing Exponentials

Suppose $X \sim \text{Exp}(\lambda_X)$, $Y \sim \text{Exp}(\lambda_Y)$, and X, Y are independent.

a. Find $P(X < Y)$.

b. Now suppose $\lambda_X = \lambda_Y = \lambda$. Using part (a), find the density of $Z = X/Y$. (Hint: look at the CDF of Z.)

c. By a similar process as in (b), find the distribution of $W = \frac{X}{X+Y}$. (Simplify F_W, and you should recognize W as one of our famous distributions.)

Problem 3

Let $X = UV$ for independent uniform $(0, 1)$ variables U and V. Find the density of X.
Ex. 5.4.9 in Pitman's Probability

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