Stat 134: Section 20

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Conceptual Review

a. What is the C.D.F. of a $Beta(r, s)$ distribution?

b. How do we find $P(Y = y)$ from $P(Y = y|X = x)$ and $P(X = x)$?

Problem 1

Suppose $U_1, U_2 \overset{\text{i.i.d.}}{\sim}$ Unif $(0, 1)$. Let $Z = Y - X$, where $X = U_{(1)}$, $Y = U_{(2)}$. Note that Z represents the range of our random variables.

a. Find the joint density $f(x, y)$ of X, Y.

b. Find the C.D.F. of Z, $F_Z(z)$.

Hint: Draw the region of interest. It may be easier to work with $P(Z \geq z)$.

c. Use part (b) to find the density of Z.

d. It can be shown that for the range $Z_n = U_{(n)} - U_{(1)}$ of n i.i.d. Unif $(0, 1)$ random variables, the CDF of Z_n is given by $F_{Z_n}(z) = z^n + nz^{n-1}(1 - z)$. Using what we know about order statistics, explain why this is the case.

Problem 2

Suppose I toss three coins. Some of them land heads and some land tails. Those that land tails I toss again. Let X be the number of heads showing after the first tossing, Y the total number showing after the second tossing, including the X heads appearing on the first tossing. So X and Y are random variables such that $0 \leq X \leq Y \leq 3$ no matter how the coins land. Write out distribution tables and sketch histograms for each of the following distributions:

a. the distribution of X;

b. the conditional distribution of Y given $X = x$ for$x$ = 0, 1, 2;

c. the joint distribution of X and Y;

d. the distribution of Y;

e. the conditional distribution of X given $Y = y$ for y = 0, 1, 2, 3.

Ex 6.1.1 in Pitman?s Probability

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