Stat 134: Section 21

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Conceptual Review

a. What is the method for finding $\mathbb{E}(Y)$ based on another random variable X?

b. What is Bayes' Rule?

c. *Random variable or constant?* For each of the following, indicate whether it is a random variable or a constant. For instance, $\mathbb{E}(X)$ would be a constant.

   (i) $\mathbb{E}(Y \mid X)$;
   (ii) $\mathbb{E}(Y \mid X = x)$;
   (iii) $\mathbb{E}(\mathbb{E}(Y|X))$;
   (iv) $\mathbb{E}(X|X)$;
   (v) $\mathbb{E}(Y \mid X = Y)$.

Problem 1

Let $X \sim$ Geom (p) on $\{1, 2, \ldots, \}$. Let $Y \sim$ Uniform $\{0, 1, \ldots, X\}$ (that is, given $X = x$, Y is uniformly distributed from 0 to x).

1. Find $\mathbb{E}(Y|X = k)$;
2. Find $\mathbb{E}(Y)$.

Problem 2

Let $X \sim$ Exponential (λ), and let $Y \sim$ Poisson (X) (that is, given $X = x$, Y follows the Pois (x) distribution).

a. Find $P(X \in dx, Y = y)$;

b. Use (a) to find the unconditional distribution of Y;

c. Given $Y = y$, what is the conditional density of X? (Hint: use Bayes' Rule).

Problem 3: Conditioning on the First Toss

Let X be the number of tosses to get heads in a coin that lands heads with probability p.

a. Argue that given the first toss is tails, the number of tosses to get heads is modeled by $1 + X^*$, where X^* and X have the same distribution.

b. Let I_1 be the indicator of whether the first toss is heads. Use part (a) and the rule $\mathbb{E}(X) = \mathbb{E}(\mathbb{E}(X|I_1))$ to show $\mathbb{E}(X) = 1/p$.

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