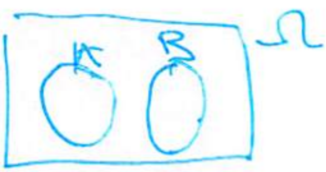
Stat 134 lec 1

Chapter 1 Intro to Probability.

To define probability we start with an <u>outcome space</u> Ω and assign to each element a nonnegative number and require that all the numbers add up to 1.

Axioms

1. $P(A) \geq 0$ for all $A \subseteq \Omega$.
2. $P(\Omega) = 1$
3. If A and B are mutually exclusive sets



then $P(A \cup B) = P(A) + P(B)$
(addition rule).

ex $\boxed{\boxed{R}\ \boxed{B}\ \boxed{G}}$ draw 2 tickets at random w/o replacement.

$\Omega = \{RB, RG, BR, BG, GR, GB\}$

$P(BR) = \frac{1}{6}$

$P(BR \cup BG) = P(BR) + P(BG) = 1/6 + 1/6 = 2/6 = 1/3$ since mutually exclusive

As a consequence of these axioms we can derive other rules.

difference rule

ex A: #72 $\quad B \subseteq A$
B: #750

$A \setminus B$ — also AB^c

difference rule $P(A\setminus B) = P(A) - P(B)$.

Pf/ $A = B \cup (A\setminus B)$ disjoint union.

$P(A) = P(B) + P(A\setminus B)$ addition rule.

$P(A\setminus B) = P(A) - P(B)$

ex $\Omega = \mathbb{Z}$ integers

$A = \# > 10$

$B = \# > 11$

$A\setminus B = \{11\}$

$P(\{11\}) = P(\# > 10) - P(\# > 11)$.

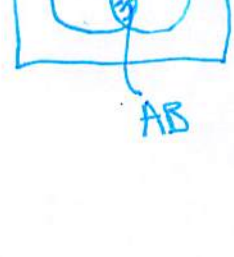
Complement rule

Apply ~~diff~~ difference rule with $A = \Omega$

$\Omega\setminus B = B^c$ ← complement.

$P(B^c) = \underbrace{P(\Omega)}_{=1} - P(B) = \boxed{1 - P(B)}$

Inclusion exclusion rule



$P(A\cup B)$

Hint: $A\cup B = A \cup \underbrace{B\setminus AB}$ disjoint union.

$P(A\cup B) = P(A) + \underbrace{P(B\setminus AB)}_{= P(B) - P(AB)}$

$= \boxed{P(A) + P(B) - P(AB)}$

Roll a die twice

$$\frac{5}{6}\cdot\frac{5}{6}=\frac{25}{36}$$

$$P(\text{at least one } 6) = 1 - P(\text{no } 6) = \frac{11}{36}$$

$$P(\text{at least one } 6) = P(\text{first roll is } 6 \text{ or second roll is } 6)$$

$$= \underbrace{P(1^{st} \text{ is } 6)}_{1/6} + \underbrace{P(2^{nd} \text{ is } 6)}_{1/6} - \underbrace{P(\text{both } 6)}_{(1/6)^2}$$

$$= 11/36$$

Book's inequality $P(A \cup B) \le P(A) + P(B)$ ← upper bound.

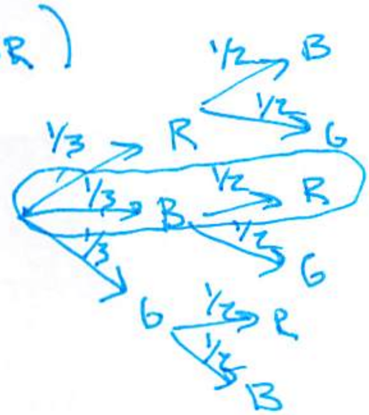
$$P(\text{at least one 6 in 2 rolls}) \le 1/6 + 1/6 = 2/6$$

$$P(\text{'' '' '' '' 5 rolls}) \le 5/6 \leftarrow \text{too big.}$$

multiplication rule

| R B G | draw 2 tickets w/o replacement

Find P(BR)



"given"

$$P(BR) = \underbrace{P(B \text{ first})}_{1/3} \cdot \underbrace{P(R\ 2^{nd} \mid B \text{ first})}_{1/2}$$

$$= 1/6.$$

mult rule $P(AB) = P(A)\cdot P(B \mid A)$

division rule $P(B \mid A) = \dfrac{P(AB)}{P(A)}$

ex (Birthday Problem)

365 days in the yr.

Each person's b-day is equally likely to be any of 365 days regardless of ~~[illegible]~~ other people's b-day.

You have a room with n people.

P(at least 2 people in room have the same b-day)

= 1 − P(no one has the same b-day).

$\underline{n=2}$ $1 - \frac{365}{365}\frac{364}{365}$

Do for general n.

$$1 - \frac{365}{365}\cdot\frac{364}{365}\cdots\frac{365-(n-1)}{365} = 1 - \prod_{i=0}^{n-1}\left(\frac{365-i}{365}\right).$$