Last time we discussed the B-day problem and found,

$$P(\text{at least 2 have the same B-day}) = 1 - \prod_{i=0}^{n-1} \frac{365-i}{365}$$

where n = number of people in the room.

Lets assume $n \le 365$ since if $n > 365$ then the answer is 1.

i-clicker question (see next page).

note: i-clickers are required for the class but wont count towards your grade. We will use clickers to keep you doing active learning in class.

i-clicker soln

let $P = \prod_{i=0}^{n-1} \left(\frac{365-i}{365}\right) = \prod_{i=0}^{n-1}\left(1 - \frac{i}{365}\right)$

$\Rightarrow \log_e P = \log_e \prod_{i=0}^{n-1}\left(1 - \frac{i}{365}\right) = \sum_{i=0}^{n-1} \log\left(1 - \frac{i}{365}\right)$

Note $\log(1+x) \sim x$ for small x.

why?

Maclauren series:

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \approx 1 + x$ for small x

$\Rightarrow \underbrace{\log_e e^x}_{= x} \approx \log_e(1+x)$ for small x.

so $\log_e P \approx \sum_{i=0}^{n-1}\left(-\frac{i}{365}\right) = -\frac{1}{365}\sum_{i=0}^{n-1} i$, $\quad \sum_{i=0}^{n-1} i = \frac{(n-1)n}{2}$

so $= -\frac{(n-1)(n)}{730}$

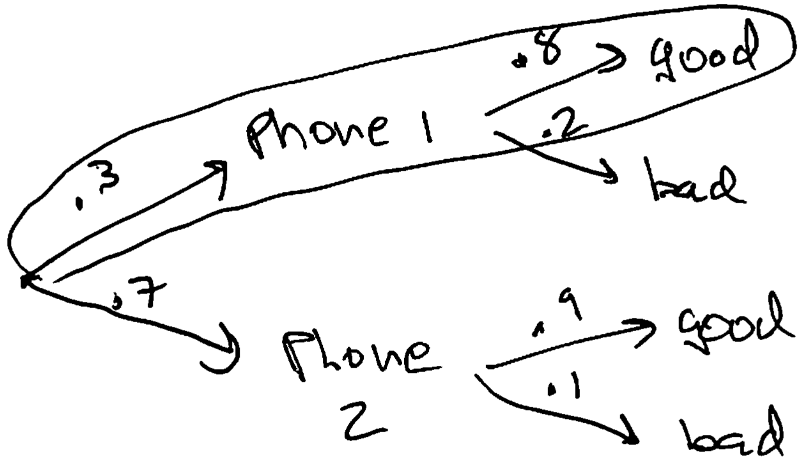
$\Rightarrow P \approx e^{-\frac{(n-1)(n)}{730}}$

$\Rightarrow P \approx e^{-\frac{(22)(23)}{730}} = 1/2$. Hence $1 - p \approx 1/2$ ← answ $\boxed{B}$

Tree Diagram and Bayes' rule

A factory produces 2 models of cell phones,

$P(\text{cell phone } 1) = .3$, $P(\text{good} \mid 1) = .8$, $P(\text{good} \mid 2) = .9$



Find

$P(1, \text{good}) = (.3)(.8)$

$P(\text{good}) =$ good = (good, 1) or (good, 2)

$P(\text{good}) = P(\text{good}, 1) + P(\text{good}, 2) = .87$

$\quad\quad = (.3)(.8) \quad\quad = (.7)(.9)$

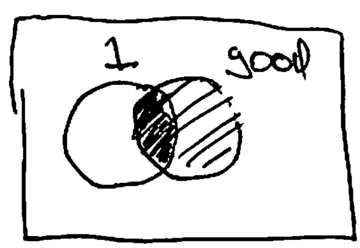
$P(1 \mid \text{good})$

Bayes rule (or division rule)

$$P(1 \mid \text{good}) = \frac{P(1, \text{good})}{P(\text{good})} = \frac{(.3)(.8)}{.87} = .28$$

Rule of finding prob backwards in time

Given the future predict the past.



Another way to write Bayes rule:

$$\boxed{P(1 \mid \text{good}) = \frac{P(\text{good} \mid 1) \cdot P(1)}{P(\text{good} \mid 1) \cdot P(1) + P(\text{good} \mid 2) \cdot P(2)}}$$

$P(1), P(2)$ are prior probabilities.

These are likely from a long run frequency interpretation (randomly pick 100 phones, 30 are type 1).

Sometimes it is impossible to give a long run freq. interpretation.

In that case my priors are subjective.

Bayes rule tells me how to update my priors given data.

$P(\text{good} \mid 1)$ is called a likelihood prob. You dont need Bayes rule to find this.

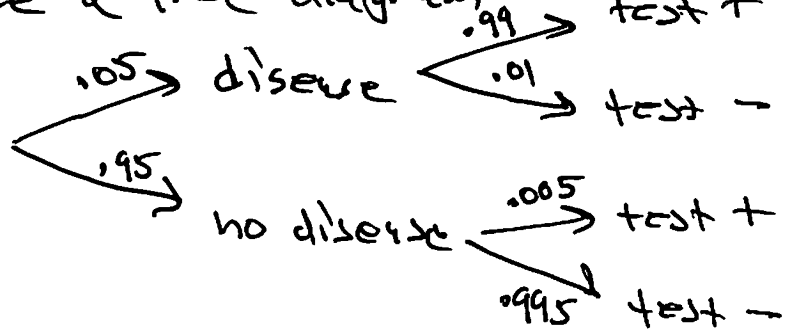
$P(1 \mid \text{good})$ is called posterior prob

$$\boxed{\text{Posterior} \underset{\text{proportional}}{\propto} \text{likelihood} \cdot \text{prior}}$$

ex 5% of population has a disease — $P(\text{disease}) = .05$
They take a test.

$$\left.\begin{array}{l} P(\text{test} + \mid \text{disease}) = .99 \\ P(\text{test} - \mid \text{no disease}) = .995 \end{array}\right\} \text{likelihood.}$$

Make a tree diagram



One person is picked at random from the population. Given test +, what is the chance he has the disease.

$$P(\text{disease} \mid \text{test } +) = \frac{P(\text{disease, test } +)}{P(\text{test } +)}$$

posterior.

$$= \frac{(.05)(.99)}{(.05)(.99) + (.95)(.005)} = .91$$

<u>law of total prob</u>

ex B_1 = disease, B_2 = no disease

Let $B_1, B_2, \ldots, B_n$ be a partition of all posible outcomes.

$$P(A) = P(A,B_1) + P(A,B_2) \cdots + P(A,B_n)$$

$$= \boxed{P(A \mid B_1)P(B_1) + \cdots + P(A \mid B_n)P(B_n)}$$