Stat 134 lec 3

Deck of cards — 4 suits H, C, D, S
13 ranks per suit Ace, 2-10, J, Q, K
52 cards.

Imagine have a 3 card deck J, Q, K

$P(2^{nd}\text{ card is }Q) = 1/3.$

$$P(Q2) = \underset{\overset{\shortparallel}{1/2}}{P(Q2|J1)}\,\underset{\overset{\shortparallel}{1/3}}{P(J1)} + \underset{\overset{\shortparallel}{1/2}}{P(Q2|K1)}\,\underset{\overset{\shortparallel}{1/3}}{P(K1)} = 1/3$$

I-clicker question (see next page).

last time <u>Bayes rule</u> $P(A|B) = \dfrac{P(AB)}{P(B)}$

ex 1.5.9

A box contains 3 shaped die, D_1, D_2, D_3, with prob $1/3, 1/2, 2/3$ respectively of landing flat (with 1, 6 on top)

fair

a) One of the 3 shapes will be chosen at random, and rolled. What is chance that the number rolled is 6?

b) Given that the number 6 is rolled, what is chance that the fair die was chosen.

a: ~~$P(D_1) = 1/3,\ P(D_2) = 1/2,\ P(D_3) = 2/3.$~~

~~$P(6)$~~

↓

(2)

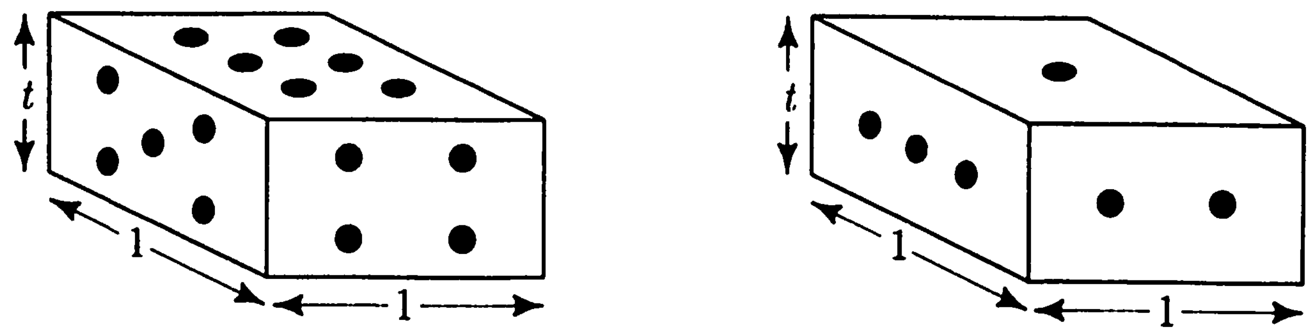
2. A deck of cards is shuffled. What is the chance that the top card is the **king** of spades **or** the bottom card is the **king** of spades

a $\frac{1}{52} \times \frac{1}{51}$
b $\frac{1}{52} + \frac{1}{51}$
c $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{51}$
d none of the above

$\frac{1}{52} + \frac{1}{52}$ (addition rule) shaped die for problem 1, 5, 9 "done" in class.

Example 3. **Shapes.**
P24

A *shape* is a 6-sided die with faces cut as shown in the following diagram:



Brain Freeze!! (4)

Now pretend we did this in class:

a) The priors are the chance of choosing any of the 3 die.

$$P(D_1) = P(D_2) = P(D_3),$$

Condition that you roll a 6 on each of the priors, we have likelihood:

$$P(\text{roll } 6 \mid D_1) = 1/6$$

$$P(\text{roll } 6 \mid D_2) = 1/4$$

$$P(\text{roll } 6 \mid D_3) = 1/3$$

prob that D_2 lands flat is $1/2$. Then chance lands flat with 6 on top is $1/2 \cdot 1/2 = 1/4$.

By law of tot prob:

$$P(\text{roll } 6) = P(\text{roll } 6 \mid D_1)\,P(D_1) + P(\text{roll } 6 \mid D_2)\cdot P(D_2) + P(\text{roll } 6 \mid D_3)\,P(D_3)$$

$$= 1/6 \cdot 1/3 + 1/4 \cdot 1/3 + 1/3 \cdot 1/3 = 1/3(1/6 + 1/4 + 1/3) = .25$$

b) $$P(D_1 \mid \text{roll } 6) = \frac{P(D_1, \text{roll } 6)}{P(\text{roll } 6)} = \frac{P(\text{roll } 6 \mid D_1)}{P(\text{roll } 6)}$$

posterior

$$= \frac{1/6 \cdot 1/3}{.25} = .222$$

this updates the prior. If you didn't know which die you had and it landed 6 you would think it is less likely to be the fair die since the other die are thinner and hence more likely to land flat and be a 6. It makes sense the posterior is less than 1/3.

Ex Sec 1.4 example 3).

There are 3 two sided cards in a box

black, white

b/b, b/w, w/w

either side can be on top.

One card is drawn

The visible side of the card is black

What is the chance the other side is white?

$P(W_{bot} \mid B_{top})$

Hint! $\Omega = \{ (b, b), (b, b), (b, w), (w, b), (w, w), (w, w) \}$

top bot

$$P(W_{bot} \mid B_{top}) = \frac{P(W_{bot}, B_{top})}{\underbrace{P(B_{top})}_{= 3/6}} = \frac{1/6}{3/6} = 1/3$$

Independence

Multiplication rule for 2 events: $P(AB) = P(A)P(B)$

3 events: $P(ABC) = P(A)P(B)P(C)$

We say A, B, C are pairwise indep, if

$P(AB) = P(A)P(B)$, $P(AC) = P(A)P(C)$ and $P(BC) = P(B)P(C)$.

Ex 3 people

B_{ij} = person i and j have same B-day.

assume each birthday equally likely.

1) Are B_{12} and B_{23} indep?

$$\underbrace{P(B_{12}B_{23})}_{= B_{123}} \stackrel{?}{=} P(B_{12})P(B_{23}) = \left(\frac{1}{365}\right)^2$$

$$B_{123} \quad \frac{365}{365}\frac{1}{365}\frac{1}{365} = \left(\frac{1}{365}\right)^2$$

2) B_{12}, B_{23}, B_{13} indep?

$$P(\underbrace{B_{12}B_{23}B_{13}}_{= B_{123} = \left(\frac{1}{365}\right)^2}) \overset{?}{\neq} P(B_{12})P(B_{23})P(B_{13}) = \left(\frac{1}{365}\right)^3$$

$\Rightarrow$ not indep.

3) B_{12}, B_{23}, B_{13} pairwise indep? ✓

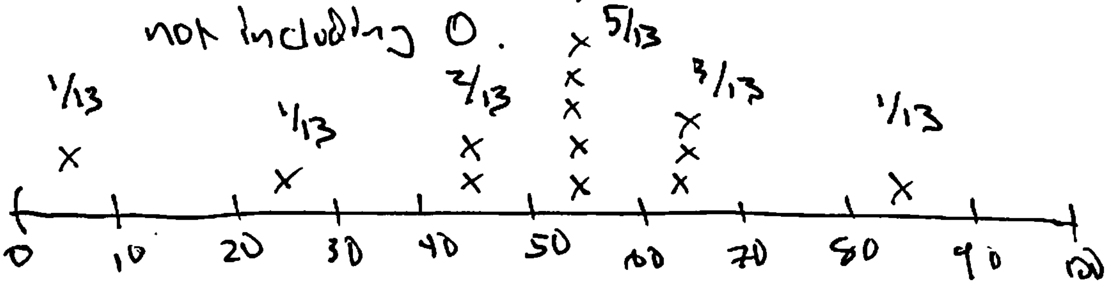
<u>Named distribution</u>

<u>empirical distribution</u> — data histogram, empirical hist.

let $$P_n(a,b) = \frac{\#\{i \overset{\text{such that}}{:} 1 \le i \le n,\ a < x_i \le b\}}{n}$$

ex test scores

I have 13 tests, scores between 0 and 100 not including 0.



ex Flip a fair coin 100 times and record # heads, Repeat experiment 1000 times.

Stat 134 lec 3

Source file ⇒ stat135-lec3.Rmd (data:text/x-markdown;base64,LS0tCnRpdGxlOiAiU3RhdCAxMzQgbGVjIDMiCmF1dGhvcjogIiKZGF0ZToglilKb3V0cHV0OiAKICBodG1sX2RvY3VtZW50OgoglCAgZmlnX2hlaV

Empirical distribution

I flip a fair coin 100 times and record the number of heads. I repeat this experiment 1000 times and plot a histogram of counts. Here is the first 500 counts.

```
##   [1] 44 50 46 63 50 49 49 50 49 49 55 56 53 48 49 55 58 45 44 50 57 49 48
##  [24] 53 44 50 46 55 52 44 48 47 51 49 55 56 48 58 53 52 49 42 51 48 50 52
##  [47] 49 41 49 56 49 46 58 47 50 52 51 46 49 55 45 47 50 52 47 47 54 50 41
##  [70] 47 50 55 52 47 52 47 50 43 43 44 49 51 55 43 52 50 45 53 48 47 48 46
##  [93] 46 49 48 47 53 42 42 47 47 49 57 46 50 42 53 51 57 42 49 46 57 57 58
## [116] 52 53 57 55 44 44 42 48 49 47 50 41 55 45 53 47 59 51 50 52 43 49 53
## [139] 57 50 47 45 52 51 46 44 49 41 48 52 53 52 56 57 48 58 60 57 50 53 52
## [162] 48 46 54 39 45 46 48 48 49 56 49 52 52 53 56 52 52 53 46 47 52 47 46
## [185] 54 42 52 44 53 50 61 46 47 46 56 45 49 52 52 50 50 53 57 52 53 48 50
## [208] 49 51 48 47 51 56 53 38 44 50 40 55 53 54 40 44 53 47 50 47 47 52 51
## [231] 53 51 45 58 51 51 48 53 51 51 51 54 58 55 54 41 50 45 53 57 49 50 44
## [254] 46 55 53 50 48 51 41 45 57 44 50 47 57 51 49 53 54 47 55 48 52 53 56
## [277] 46 52 58 55 58 43 59 49 48 52 56 39 50 50 48 51 48 52 47 47 61 47 48
## [300] 42 51 52 51 58 46 41 46 47 46 52 57 47 43 53 43 49 48 51 57 56 49 57
## [323] 45 55 52 37 49 47 45 52 49 55 47 51 49 44 52 48 48 47 59 51 51 54 41
## [346] 57 51 43 50 54 52 49 46 57 56 54 48 46 54 47 55 56 60 49 61 52 51 48
## [369] 49 52 50 46 47 43 62 59 55 50 57 46 57 49 48 54 56 51 52 46 55 58 48
## [392] 43 48 56 52 54 49 52 45 43 54 44 50 48 52 50 48 45 52 51 48 47 58 48
## [415] 44 52 42 50 50 51 48 51 44 45 58 40 48 55 46 47 53 54 44 50 52 49 51
## [438] 51 45 53 54 46 52 60 49 44 50 53 49 47 45 54 51 50 46 57 55 52 50 56
## [461] 53 53 48 50 44 58 37 49 51 55 49 58 52 56 49 48 56 53 51 55 48 44 48
## [484] 50 51 48 52 56 48 55 53 48 54 45 48 58 45 50 50 58
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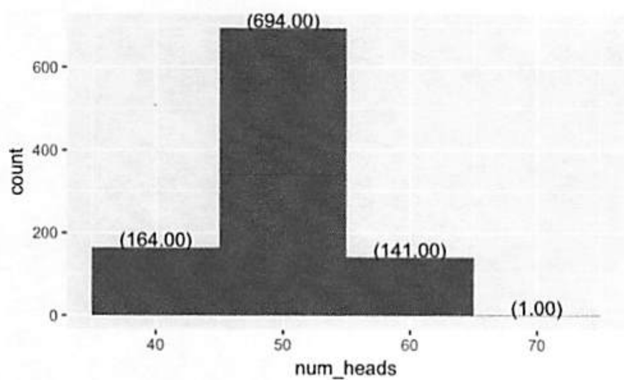

Dividing the counts by 1000 we get the emphirical distribution.