Sampling without renkelement

Hypergeometric distribution

$$\text{ex} \quad P(\text{Poker hand has 2 aces}) = g \frac{\binom{4}{2}\binom{48}{34}}{\binom{52}{54}} b$$

where G, B, N, n labels the terms.

$\uparrow g$

Hypergeometric (N, G, n)

$N = \text{pop size} \leftarrow G + B$

$n = \text{SRS size} \leftarrow g + b$

$G = \text{good in pop}$

$B = \text{bad in pop}$

$$P(g \text{ good elements in sample}) = \frac{\binom{G}{g}\binom{B}{b}}{\binom{N}{n}}$$

*Binomial is distribution of successes w/ renkelement

Hypergeom " " " " " w/o "

ex (2.5.2) 3 cards dealt from standard deck.

w/ 26 red, 26 black.

Find $P(1^{st}\text{ red } \underline{and}\text{ second two are black})$.

$= P(\text{second two are black} | 1^{st}\text{ red}) \cdot P(1^{st}\text{ red})$ mult rule

$\qquad\qquad = \qquad\qquad\qquad\qquad\qquad = \frac{1}{2}$

$$\frac{\binom{26}{2}\binom{25}{0}}{\binom{51}{2}}$$

$$= \boxed{\frac{26 \cdot 25}{51 \cdot 50} \cdot \frac{1}{2}}$$

ex (2.5.1)

Suppose you take a random sample of 10 tickets w/o replacement from a box containing 20 red tickets and 30 blue tickets.

a) Find P(get exactly 4 red tickets)

$$\frac{\binom{20}{4}\binom{30}{6}}{\binom{50}{10}} = .28 \quad \leftarrow \text{in R.}$$

b) What is answer if draw w/ replacement.

$n = 10$

$P = 20/50 = 2/5$

$k = 4$

$$\binom{10}{4}\left(\tfrac{2}{5}\right)^4\left(\tfrac{3}{5}\right)^6 = .25$$

Slot method

R R R R B B B B B B

$$\frac{20}{50}\,\frac{19}{49}\,\frac{18}{46}\,\frac{17}{47}\,\frac{30}{46}\,\frac{29}{45}\,\frac{28}{44}\,\frac{27}{43}\,\frac{26}{42}\,\frac{25}{41} = \frac{\frac{20!}{16!}\cdot\frac{30!}{24!}}{\frac{50!}{40!}}$$

All orderings of 4R and 6B is $\binom{10}{4} = \frac{10!}{4!6!}$

$$\Rightarrow \quad \frac{10!}{4!6!}\cdot\frac{\frac{20!}{16!}\cdot\frac{30!}{24!}}{\frac{50!}{40!}} = \frac{\frac{20!}{16!4!}\cdot\frac{30!}{24!6!}}{\frac{50!}{40!40!}} = \frac{\binom{20}{4}\binom{30}{6}}{\binom{50}{10}}$$

When $n \ll N$ Hypergeom $(N, G, n) \to Bin\left(n, \frac{G}{N}\right)$

Sampling distribution

$$\boxed{100 R \quad 100 B}$$

$\downarrow$ draw n

Count #R in sample

$\leftarrow$ sampling distribution.

Red

$\dfrac{n}{2}$
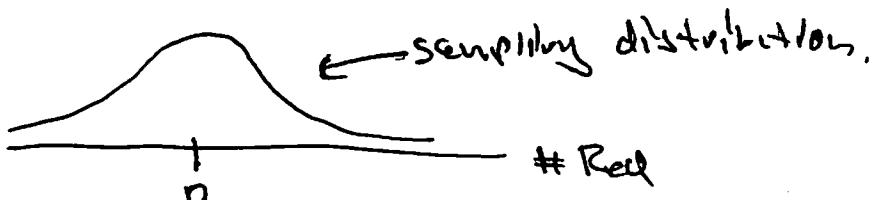
ex: I-clicker question (see next page)

$\underline{\underline{ex}}$ 5 card poker hand

ranks $aaaa\,b$, $a \neq b$

$P($ four of a kind $)$

Choose method $\quad \dfrac{\dbinom{13}{1}\dbinom{12}{1}\dbinom{4}{4}\dbinom{4}{1}\dbinom{44}{0}}{\dbinom{52}{5}} = \boxed{.00024}$

not $\dbinom{13}{2}$ since $aaaab \neq bbbba$.

Slot method $\quad \dbinom{5}{1}\dfrac{52}{52}\dfrac{3}{51}\dfrac{2}{50}\dfrac{1}{49}\dfrac{48}{48} = \boxed{.00024}$ ✓

try with slot method to see see get same answer.

$P($ full house $)$ — ranks $aaa\,bb$.

Choose method $\quad \dfrac{\dbinom{13}{1}\dbinom{12}{1}\dbinom{4}{3}\dbinom{4}{2}}{\dbinom{52}{5}} = .00144$

Slot method $\dbinom{5}{2}\dfrac{52}{52}\dfrac{3}{51}\dfrac{2}{50}\dfrac{48}{49}\dfrac{3}{48} = \boxed{.00144}$ ✓

$P($ two pairs $)$ — ranks $aa\,bb\,c$ $\qquad a \neq b \neq c$

Choose method $\quad \dfrac{\dbinom{13}{2}\dbinom{11}{1}\dbinom{4}{2}\dbinom{4}{2}\dbinom{4}{1}}{\dbinom{52}{5}} = \boxed{.0475}$

not $\dbinom{13}{1}\dbinom{12}{1}$ since $aabbc = bbaac$.

Slot method $\dbinom{5}{1}\dfrac{52}{52}\dfrac{3}{51}\dfrac{48}{50}\dfrac{3}{49}\dfrac{44}{48} = \boxed{.0475}$ ✓

$\nwarrow$ ways to choose 1 single

Chapter 2   Friday February 2 2018

1. A box has 100 red and 100 blue marbles. You randomly draw $n \leq 200$ marbles and count the number or red in your sample. In which case will the sampling distribution of counts be narrower?

   a sample with replacement

   b sample without replacement

in chap 3 we will see

w/replacement

correction factor

$$\sigma \left(\sqrt{\frac{N-n}{N-1}} \right)$$

$N \to \infty$

σ

$\frac{n}{2}$ #R

$\frac{n}{2}$ #R