Stat 134 lec 10

Chap 3 Random Variables,

A prob space consists of an outcome space Ω and a prob function P satisfying 3 axioms ($P(A) \geq 0$, $P(\Omega) = 1$, add^n rule).

Random variable X is a function from $\Omega \to \mathbb{R}$.

$$\omega \to X(\omega)$$

ex flip a fair coin 2 times

X = # heads you get — 0, 1, 2

$\Omega = \{HH, HT, TH, TT\}$

$X : \Omega \to \mathbb{R}$

$HH \to 2$

$$\{X=1\} = \{\omega : X(\omega) = 1\} = \{HT, TH\}$$

$$P(X=1) = \sum_{\omega \in \{X=1\}} P(\omega) = \underset{\frac{1}{4}}{P(HT)} + \underset{\frac{1}{4}}{P(TH)} = 1/2$$

$$P(X > 0) = \sum_{x=1}^{2} P(X=x) = P(X=1) + P(X=2)$$

We dont write $P(X)$

Distribution of X

x	0	1	2
$P(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
$\lvert x-1 \rvert$	1	0	1

$X \sim Bin(2, \frac{1}{2})$

Find the distribution of $|X-1|$

$\lvert x-1\rvert$	0	1
$P(\lvert x-1\rvert)$	$\frac{1}{2}$	$\frac{1}{2}$

$|X-1| \sim Bin(1, \frac{1}{2})$

Joint Distributions

X, Y are RV on same outcome space Ω — this might be complicated.

The joint distribution (X, Y) has values $(x \in X, y \in Y)$ with probability $P(X=x, Y=y)$

ex X: one draw from $\boxed{1}\boxed{2}\boxed{2}\boxed{3}$

Given $X=x$, Y = number of heads in x coin tosses. (1, 2, 3 ; 0, 1, 2, 3)

The joint distribution is the prob of all $(x \in X, y \in Y)$.

$$P(X=1, Y=1) = \underbrace{P(Y=1 \mid X=1)}_{=\frac{1}{2}} \cdot \underbrace{P(X=1)}_{=\frac{1}{4}} \quad \text{mult rule}$$

Y \ X	1	2	3	marginal Y
3	0	0	$\frac{1}{8}\cdot\frac{1}{4}$	.031
2	0	$\frac{1}{4}\cdot\frac{1}{2}$	$\frac{3}{8}\cdot\frac{1}{4}$	.219
1	$\frac{1}{2}\cdot\frac{1}{4}$	$\frac{1}{2}\cdot\frac{1}{2}$	$\frac{3}{8}\cdot\frac{1}{4}$	.469
0	$\frac{1}{2}\cdot\frac{1}{4}$	$\frac{1}{4}\cdot\frac{1}{2}$	$\frac{1}{8}\cdot\frac{1}{4}$	.281
marginal X	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	1

Marginal prob $P(X=x) = \sum_{\text{all } y} P(X=x, Y=y)$

$\underbrace{x}_{\text{fixed}}$ $\quad$ $\underbrace{X=x}_{\text{fixed}}$, $\underbrace{Y=y}_{\text{varies}}$

Conditional Dist of X given Y $\quad X|Y$

By mult rule

$$P(X=x, Y=y) = P(X=x \mid Y=y) \cdot P(Y=y)$$

Hence

$$\boxed{P(X=x \mid Y=y) = \frac{P(X=x, Y=y)}{P(Y=y)}}$$ Baye's Rule.

Table of $P(X=x \mid Y=y)$:

	$X = 1$	2	3
$P(X=x \mid Y=3)$	0	0	$\frac{\frac{1}{8}\cdot\frac{1}{4}}{.031} = 1$
$P(X=x \mid Y=2)$	0	.572	.428
$P(X=x \mid Y=1)$	.267	.533	.200
$P(X=x \mid Y=0$	.444	.444	.111

Do values of this table change from row to row? Yes

$\Rightarrow$ X, Y are dependent random variables, Two RV are <u>independent</u> if

$$P(X=x \mid Y=y) = P(X=x)$$

By mult rule
If X, Y indep.

$$\boxed{P(X=x, Y=y) = \underbrace{P(X=x \mid Y=y)}_{= P(X=x)} P(Y=y)}$$

"iid" – ~~the~~ independent and identically distributed

X, Y are iid if they are independent and

X, and Y have the same range and for every value v in the range $P(X=v) = P(Y=v)$. — identically distributed.

ex

| 1 2 3 |

draw 2 times w/o replacement

X = 1st draw
Y = 2nd draw.

Pos values for X, Y — both 1, 2, 3

$$P(X=1) \stackrel{?}{=} P(Y=1)$$
$$P(X=1) = \tfrac{1}{3}, \quad P(Y=1) = \tfrac{1}{3}$$

but X, Y not independent.

Sum of Two RV

ex $X \sim \text{Pois}(\mu)$ — $0, 1, 2, 3, \ldots$

$Y \sim \text{Pois}(\lambda)$ — $0, 1, 2, 3, \ldots$

$S = X + Y$ Find dist of S. — pos values $0, 1, 2, 3, \ldots$

For $s \geq 0$

Find $P(S = s)$.

Pick up next time.