# Stat 134

**Chapter 3 Monday February 12 2018**

1. Consider a well shuffled deck of cards. The expected number of cards before the first ace is?

   **a** 52/5

   **b** 48/5

   **c** 48/4

   **d** none of the above

X = # cards before $1^{st}$ ace

$A_1, A_2, \ldots, A_{48}$

$1^{st}$ nA before $1^{st}$ ace

$P(A_1) = 1/5$ $\quad E(X) = \underset{1/5}{P(A_1)} + \cdots + \underset{1/5}{P(A_{48})} = 48/5$

$P(A_2) = 1/5$

Answer to Jonny's question: "Why isn't the answer $\frac{48}{4} = 12$?"

Answer: Think of the deck like this

— $A_1$ — $A_2$ — $A_3$ — $A_4$ —.

You have 48 nonaces we have to fit in 5 sections. Hence $48/5$. The answer $48/4$ assumes the last card is an ace.

## Property of Expectation

For $X \ge 0$ mean for all $\omega \in \Omega$, $X(\omega) \ge 0$

$X \ge Y$ mean for all $\omega \in \Omega$, $X(\omega) \ge Y(\omega)$

$X, Y$ defined on same outcome space.

$$X(\omega) \ge Y(\omega)$$

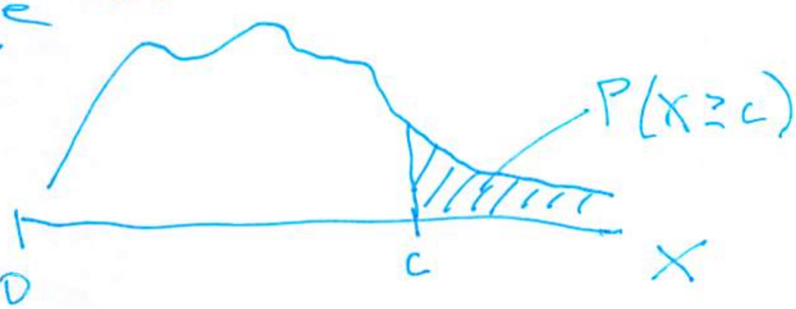
$$X(\omega)P(\omega) \ge Y(\omega)P(\omega)$$

Sum over all $\omega \in \Omega$

$$E(X) \ge E(Y).$$

## Markov inequality:

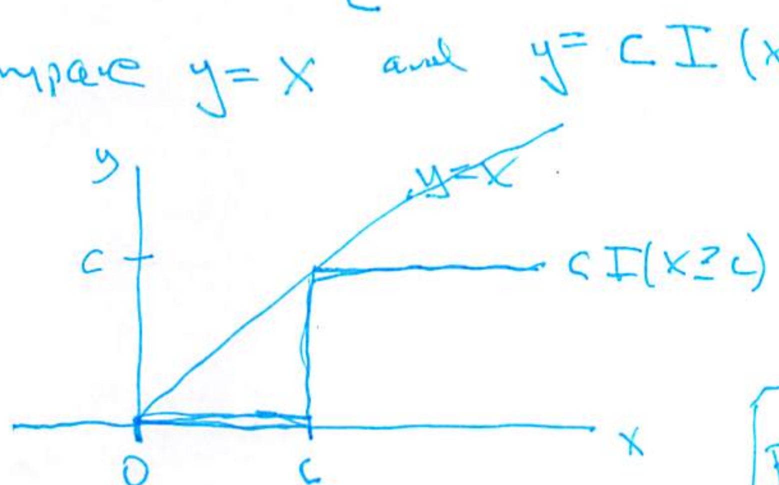
Picture $X \ge 0$




Assume $X \ge 0$ and $c$ a positive constant.

let $I(X \ge c) = \begin{cases} 1 & \text{if } X \ge c \\ 0 & \text{else.} \end{cases}$

compare $y = X$ and $y = c\,I(X \ge c)$

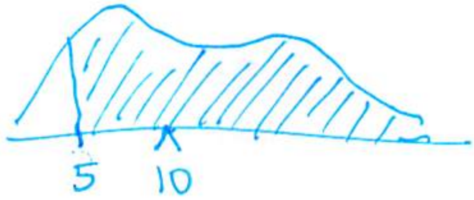



$$X \ge c\,I(X \ge c)$$

$$E(X) \ge c\,\underbrace{E(I(X \ge c))}_{= P(X \ge c)}$$

$$\boxed{P(X \ge c) \le \frac{E(X)}{c}}$$ Markov inequality

ex $E(X) = 10$
$c = 5$




$P(X \geq 5) = \frac{10}{5} = 2.$

Markov not useful if $c \leq E(X)$

goodnews This is true for any ~~positive~~ nonneg distribution if know $E(X)$

badnews need $X \geq 0$.

ex (3.2.4)
Suppose all the numbers in a list of 100 numbers are nonnegative, and the avg of the list is 2.
Prove that ~~at least~~ $\geq$ 25 numbers in the list are greater ~~than~~ or equal to 8.

$X$ = a number on the list, $X \geq 0$
$E(X) = 2$
By Markov $P(X \geq 8) \leq \frac{E(X)}{8} = 1/4$

$\frac{100}{4} = 25$

Tail sum formula. — use when calculating $P(X \geq x)$ is easier to find than $P(X = x)$.
$X$ must be a count of events.

$X$ values $0, 1, 2, 3 \ldots$
$p_0 \; p_1 \; p_2 \; p_3 \ldots \qquad 0 \leq p_i \leq 1$

$$E(X) = \begin{array}{lll} p_1 & & \\ p_2 + & p_2 & \\ p_3 + & p_3 + & p_3 \\ \vdots & \vdots & \vdots \end{array} \quad \begin{array}{l} \leftarrow P(X \geq 1) \\ \leftarrow P(X \geq 2) \\ \leftarrow P(X \geq 3) \end{array}$$

$$\Rightarrow \boxed{E(X) = P(X \geq 1) + P(X \geq 2) + \cdots}$$

ex Roll a die 3 times.

$X = \min(X_1, X_2, X_3)$

1,2,3,4,5,6

$$P(X \geq x) = P(X_1 \geq x, X_2 \geq x, X_3 \geq x) = P(X_1 \geq x)\,P(X_2 \geq x)\,P(X_3 \geq x)$$
$$= P(X_1 \geq x)^3$$

$P(X_1 \geq 1) = 6/6$

$P(X_1 \geq 2) = 5/6$

$\vdots$

$P(X_1 \geq 6) = 1/6$

$E(I_{X \geq 1})$

By Tail Sum formula

$$E(X) = P(X \geq 1) + P(X \geq 2) + \dots P(X \geq 6)$$

$$P(X \geq 1) = P(X_1 \geq 1)^3 = (6/6)^3, \quad P(X \geq 2) = (5/6)^3, \quad P(X \geq 6) = (1/6)^3$$

$$= \boxed{\frac{1}{6^3}\left(1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3\right)}$$

<u>Advice</u> when computing expectations.

- avoid computing $E(X)$ using the definition unless you have few terms.
- Is $X$ a named distribution ~~the~~ that you know the expectation of.
- If $X$ is counts of events use indicators or tail sums.

## SD and Normal Approx

$X - E(x)$ — deviation from the mean.

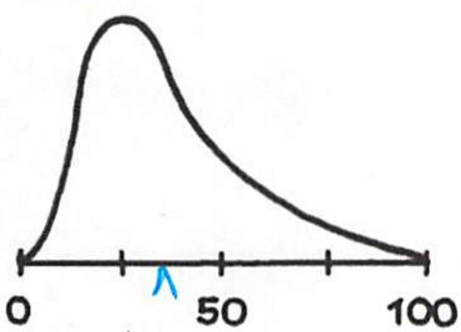
$$E(x - E(x)) = E(x) - \underbrace{E(E(x))}_{= E(x)} = 0$$

$(x - E(x))^2$ square deviation

Variance $Var(x) = E\left((x - E(x))^2\right)$

Standard deviation $= SD(x) = \sqrt{Var(x)} = \sqrt{E\left((x-E(x))^2\right)}$

~~Proof~~




avg = 35

SD = 5, 15, 50 ?

think of SD as the average spread from the center of your data.