Sec 3.5 Poisson Dist

$X \sim Pois(\mu)$

$P(X=k) = \frac{e^{-\mu}\mu^k}{k!} \quad k=0,1,2,3,\ldots$

We showed earlier

$\left.\begin{array}{l} X \sim Pois(\mu) \\ Y \sim Pois(\lambda) \end{array}\right\}$ indep $\quad S = X+Y \sim Pois(\mu+\lambda)$

$\Leftarrow \left.\begin{array}{l} X \sim Bin(n,p) \\ Y \sim Bin(m,p) \end{array}\right\}$ indep.

$X+Y \sim Bin(n+m,p) \to Pois(np+mp)$

$$E(X) = \sum_{k=0}^{\infty} k \cdot e^{-\mu}\frac{\mu^k}{k!} = \sum_{k=1}^{\infty} k e^{-\mu}\frac{\mu^k}{k!}$$

$\mu^k = \mu \cdot \mu^{k-1}$

$k! = k\cdot(k-1)!$

$$= \mu \cancel{e^{-\mu}} \underbrace{\sum_{k=1}^{\infty}\frac{\mu^{k-1}}{(k-1)!}}_{\underbrace{1+\mu+\frac{\mu^2}{2!}+\frac{\mu^3}{3!}+\cdots}_{= \cancel{e^{\mu}}}} = \boxed{\mu}$$

Trick to find $Var(X)$

$$Var(X) = E(X^2) - E(X)^2$$

$$= \underbrace{E(X^2) - E(X)}_{= E(X^2 - X) = E(X(X-1))} + E(X) - E(X)^2$$

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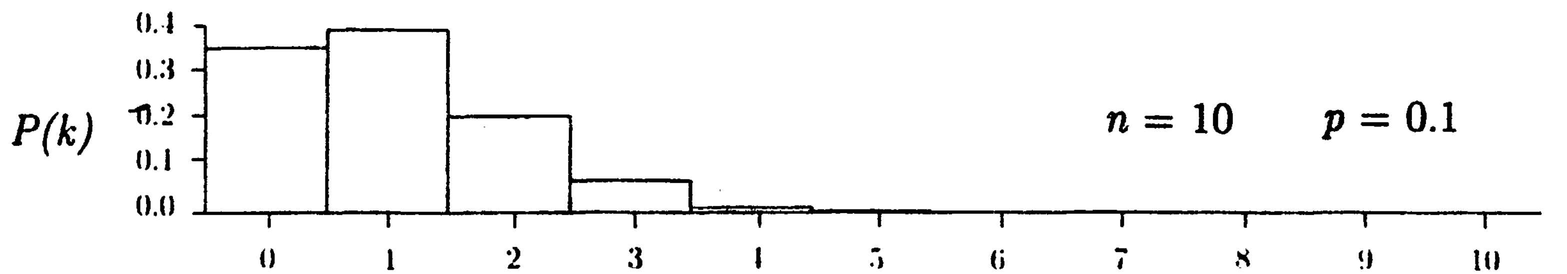
Example 1. **The binomial (10, 1/10) distribution.**

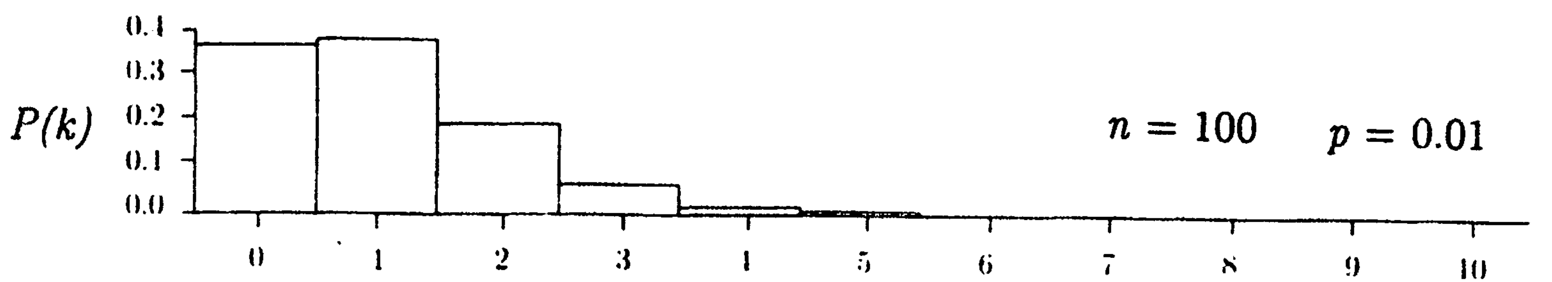
This is the distribution of the number of black balls obtained in 10 random draws with replacement from a box containing 1 black ball and 9 white ones.

Poisson($\mu=1$) is a limit of binomials Bin($n, \frac{1}{n}$)



Example 2. **The binomial (100, 1/100) distribution.**

This is the distribution of the number of black balls obtained in 100 random draws with replacement from a box containing 1 black ball and 99 white ones.

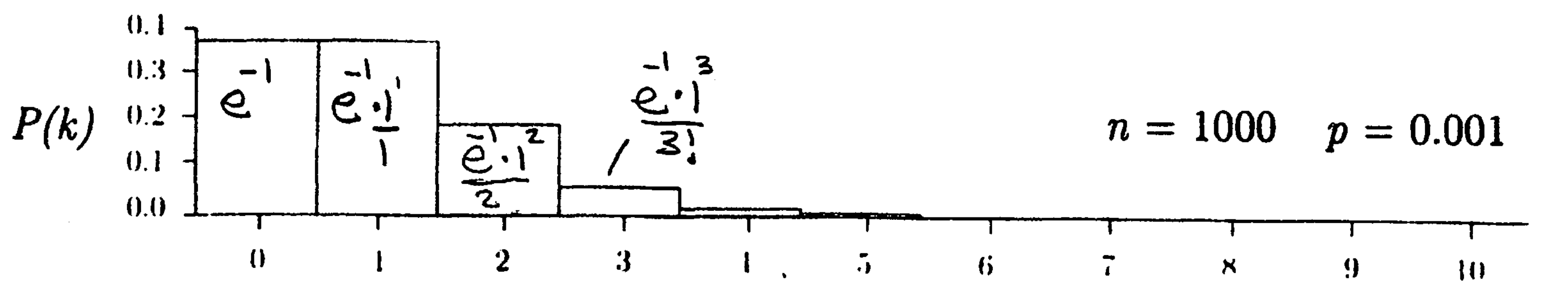


Example 3. **The binomial (1000, 1/1000) distribution.**

Now take 1000 random draws with replacement from a box with 1 black ball and 999 white ones. This is the distribution of the number of black balls drawn:

$X \sim \text{Pois}(1)$

$P(X=k) = \frac{e^{-1}}{k!}$



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Recall

$$E(g(X)) = \sum_{\text{all } x} g(x) P(X=x)$$

Exam

$$E(X(X-1)) = \sum_{k=0}^{\infty} \cancel{k}(\cancel{k-1}) P(X=k)$$

$$P(X=k) = \frac{e^{-\mu}\mu^k}{k!}, \quad \mu^k = \mu^2 \cdot \mu^{k-2}, \quad k! = \cancel{k}(\cancel{k-1})(k-2)!$$

$$= \sum_{k=2}^{\infty} e^{-\mu}\mu^2 \frac{\mu^{k-2}}{(k-2)!}$$

$$= \mu^2 e^{-\mu} \underbrace{\sum_{k=2}^{\infty} \frac{\mu^{k-2}}{(k-2)!}}_{= e^{\mu}} = \boxed{\mu^2}$$

$1 + \mu + \frac{\mu^2}{2!} + \dots$

$Var(X) = \mu^2 + \mu - \mu^2 = \boxed{\mu}$ ← mean = var for poisson!

$\boxed{SD(X) = \sqrt{\mu}}$

This makes sense if $X \sim Bin(n,p)$ w/ p small, $q \approx 1$

$$SD(X) = \sqrt{\underbrace{np}_{=\mu}\underbrace{q}_{\approx 1}} \approx \sqrt{\mu}.$$

I-clicker - see next page.

Poisson Random Scatter (PRS)

Thinking of Poisson(μ) dist. as the limit of $Bin(n,p)$ for $n \to \infty$, $p \to 0$, $np \to \mu$, we see that Pois(μ) can be used to model counts of low probability independent events.

2.5

Stat 134

Friday February 23 2018

1. Which of the following can be modeled as a Poisson Random Scatter with intensity $\lambda > 0$?

   a The number of blueberries in a 3 cubic inch blueberry muffin

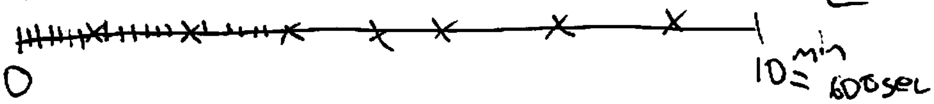
   b The number of customers entering a bank in a 24 hour period. — no customers outside of banking hours

   c The number of times a day a person feels hungry — not independent hits

   d The number of cigarettes smoked by Professor Lucas per day — $\lambda = 0$ since I don't smoke

   e more than one of the above

ex The number of calls coming into a hotel reservation center in 10 minutes, Say $\mu = 5$.



← distribution of calls should look random not clustered.

idea divide 10 min into small intervals (say every second)

PRS assumptions

1) No time interval gets more than 1 call,

each interval is a Bernoulli (p) trial, getting a call is success, n is large so chance a call lands in an interval is small (p small),

2) Have independent Bernoulli trials (i.e calls are indep of each other).

The mean number of calls in 10 minutes is $\underset{5}{\mu} = np$

let $\lambda = \mu/10$ be the intensity (rate) of calls/min for our PRS.

↑ or some unit of time.

ex Blueberry muffin

PRS intensity $\lambda = 2$ blueberries per cubic inch

A muffin is 3 cubic inches

On average how many blueberries per muffin

$\mu = \lambda \cdot 3 = 6$ blueberries.

Another muffin (from same large tub of batter)
size 4 cubic inches.

let X_2 = # blueberries in second muffin

$X_2 \sim Pois(8)$

Find P(5 blueberries in each muffin)

$= P(X_1 = 5, X_2 = 5) = P(X_1=5)P(X_2=5)$

$= \boxed{\frac{e^{-6} 6^5}{5!} \cdot \frac{e^{-8} 8^5}{5!}}$

Find P(10 blueberries total in both muffins)

$P(X_1 + X_2 = 10) = \boxed{\frac{e^{-14} 14^{10}}{10!}}$