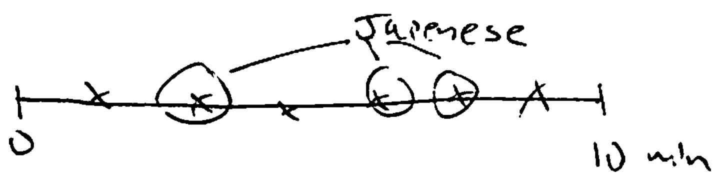
Poisson Thinning

Cars arrive at a toll booth according to Poisson process (i.e PRS) at a rate $\lambda = 3$ arrivals/min for 10 minutes.

(i.e X = # cars arriving at a toll booth in 10 min
$X \sim \text{Pois}(30)$).



Of the cars arriving, it is known over the long term 60% are Japanese imports.

Call Japanese cars a success and non Japanese cars a failure.
Each hit is a success w/ prob $p = .6$ independent of all other hits.
Then the process of "success" hits in your PRS is a PRS with intensity λp
and the process of "failure" hits in your PRS is an independent PRS w/ intensity λq.

ex What is prob that in a given 10 min interval, 15 cars arrive at the booth and 10 are Jap imports?

X = # cars in 10 min $\sim \text{Pois}(30)$
J = # Japenese cars in 10 min $\sim \text{Pois}(3(.6)\cdot 10) = \text{Pois}(18)$
nJ = # non Jap cars in 10 min $\sim \text{Pois}(12)$

We assume J and nJ are indep $\Rightarrow J + nJ = X$

$$P(X=15, J=10) = P(nJ=5, J=10)$$
$$= P(nJ=5)\cdot P(J=10)$$
$$= \boxed{\frac{e^{-12}\,12^{5}}{5!}\cdot\frac{e^{-18}\cdot 18^{10}}{10!}}$$

left over topic from 3.4 :

<u>Min of indep Geom RVs</u>

let $X \sim \text{Geom}(p_1)$, $Y \sim \text{Geom}(p_2)$ indep

$P(X>k) = q_1^k$, $P(Y>k) = q_2^k$

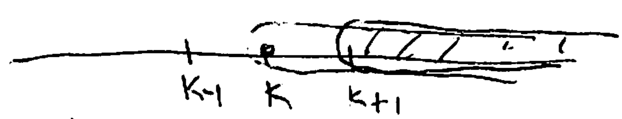
If you want to find the distribution of $W = \min(X,Y)$.

<u>trick</u> $P(W>k) = P(X>k, Y>k)$

$$= P(X>k)P(Y>k) = q_1^k \cdot q_2^k = (q_1q_2)^k$$

This defines the distribution of W since

$P(W=k)$



$$= P(W>k-1) - P(W>k)$$
$$= (q_1q_2)^{k-1} - (q_1q_2)^k$$
$$= \underbrace{(q_1q_2)}_{q}^{k-1}\underbrace{(1-q_1q_2)}_{= p}$$

Stat 134

Monday February 26 2018

1. If X$\sim$Geom(p_1) and Y$\sim$Geom(p_2) are independent then which of the following statements is true about W=min(X,Y)?

   a W$\sim$Geom($1 - p_1 p_2$)

   b W$\sim$Geom($q_1 q_2$)

   c W$\sim$Geom($1 - q_1 q_2$)

   d None of the above

Review

Finding expectation

Methods

ex 3.2.1 — 1) Definition $E(X) = \sum_{\text{all } x} x P(X=x)$ — only as last resort unless # outcomes is small

ex 3.2.14 — 2) indicators $X = I_{A_1} + \cdots + I_{A_n}$ — when X is a count of events that occur some collection of events $A_1, A_2, \ldots, A_n$

$E(X) = P(A_1) + \cdots + P(A_n)$

ex 3.2.13c — 3) Tail Sums $E(X) = P(X \geq 1) + P(X \geq 2) + \cdots$ — when X has values $X = 0, 1, 2, \ldots$ and you know tail probs,

4) recognize the distribution of X and use ~~the~~ known $E(X)$

ex $X \sim Bin(n,p)$
$E(X) = np$.

5) Write X as a sum of simpler RVs.

ex coupon collector problem from Dec 15 — ex $X = X_1 + \cdots + X_n$ where $X_i \sim Geom(p_i)$

6) Other

3.2.16

4

ex Flip 2 fair coins until you get your first heads. What is the ~~___~~ expected # of coin tosses?

$X, Y \overset{iid}{\sim} Geom(\frac{1}{2})$

$W = \min(X, Y) \sim Geom(1 - \frac{1}{2}\frac{1}{2}) = Geom(3/4)$

$E(W) = \frac{1}{3/4} = 4/3$

ex

coin 1	coin 2
T	T
T	T
T	H
H	

Here we must flip coins $3 = \min(4, 3)$ times until first H.

ex There are 10 students preparing for a TV show audition. A student has one of 3 possible skills.

a) sing only – 5
b) dance only – 2
c) sing and dance – 3

} each student has exactly one of those skills.

If 5 students are randomly selected how many different skills do you expect the 5 students will have?

1, 2, 3 ← X = the # diff skills 5 students have

$X = I_{A_1} + I_{A_2} + I_{A_3}$ where A_1 = at least 1 student can sing only

A_2

A_3

$P(A_1) = 1 - P(\text{no one can sing only})$ — $\frac{\binom{5}{0}\binom{5}{5}}{\binom{10}{5}}$ ← sing only + not sing only

$P(A_2) = 1 - P(\text{no one can dance only})$ — $\frac{\binom{2}{0}\binom{8}{5}}{\binom{10}{5}}$

$P(A_3) = 1 - P(\text{no one can dance and sing})$ — $\frac{\binom{3}{0}\binom{7}{5}}{\binom{10}{5}}$