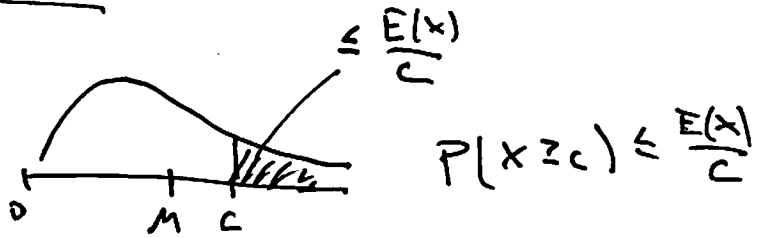
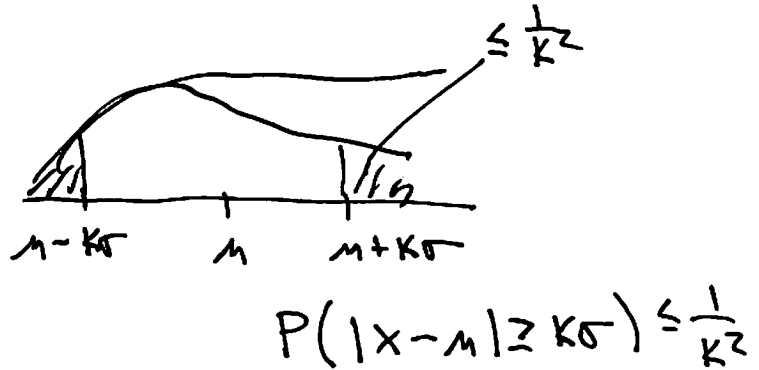


Markov and Chebyshev's Ineq

Markov Ineq  $X \geq 0$   
w/  $\mu = E(X)$



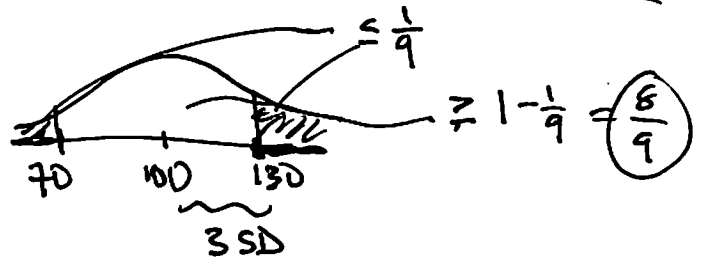
Chebyshev Ineq  
any  $X$   
 $\mu = E(X)$   
 $\sigma = SD(X)$



ex Let  $X$  be a RV s.t.  $E(X) = 100 = \text{var}(X)$ .

a) Can you find  $E(X^2)$  exactly? If not, is there anything you can tell me about it that is more informative than "non-negative".  
 $\text{var}(X) = E(X^2) - E(X)^2$   
 $E(X^2) = \text{var}(X) + E(X)^2 = \boxed{100 + 10000}$

b) Can you find  $P(70 < X < 130)$  exactly? If not is there anything you can tell me more informative than between 0 and 1.



ex  $X$  is RV w/  $E(X) = 3$  and  $SD(X) = 2$ .

T or F (justify)  $P(X^2 \geq 40) \leq \frac{1}{3}$ .

$X^2 \geq 0$   
 $E(X^2) = \text{var}(X) + E(X)^2 = 4 + 9 = 13$

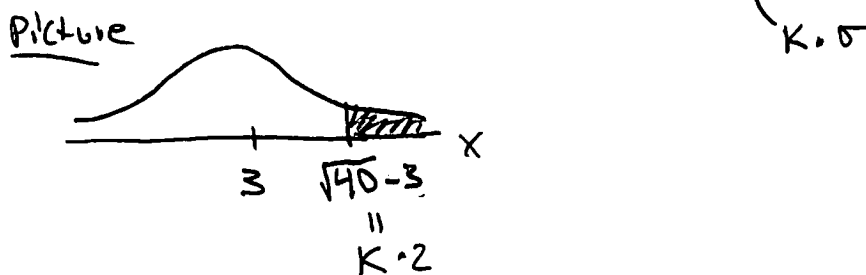
$P(X^2 \geq 40) \leq \frac{13}{40} < \frac{13}{39} = \frac{1}{3}$   
 Markov

True

You may also try this with Chebyshev:

(2)

$$P(X^2 \geq 40) = P(X \geq \sqrt{40}) = P(X - 3 \geq \underbrace{\sqrt{40} - 3}_{k \cdot \sigma})$$



$$\sqrt{40} - 3 = k \cdot \sigma$$

$$\Rightarrow k = \frac{\sqrt{40} - 3}{\sigma} = 1.66$$

Chebyshev gives

$$P(X - 3 \geq 1.66(\sigma)) \leq \left(\frac{1}{1.66}\right)^2 = \frac{1}{2.75}$$

so Chebyshev doesn't show it is less than  $\frac{1}{3}$  but Markov does.

# Instructions

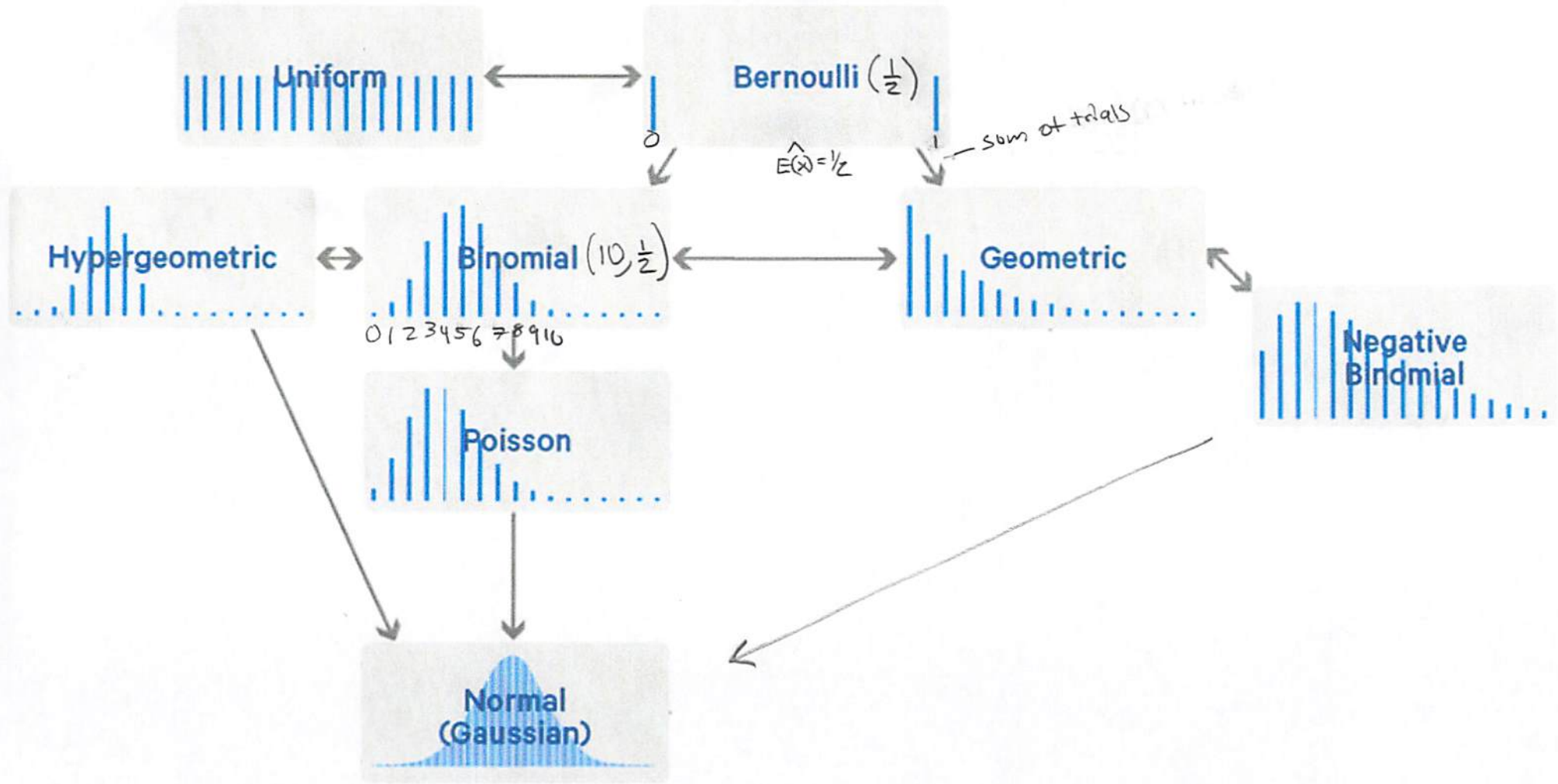
Next to each distribution:

- 1) Write parameter to match histogram
- 2) Describe a RV,  $X$ , with that histogram
- 3) Give values of  $X$
- 4) indicate  $E(X)$  and its value in histogram
- 5) briefly explain relationship given by arrow

# Distribution Summary

3

$-0, 1$   
 $X = \# \text{ heads in one coin toss of fair coin}$



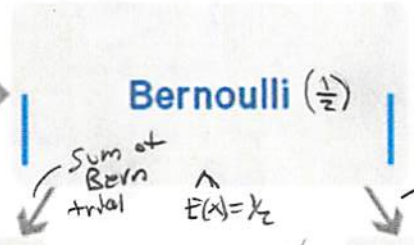
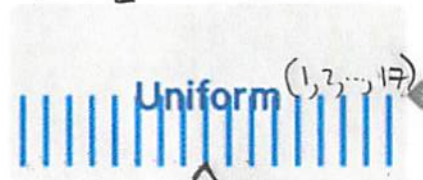
Possible solution?

(4)

1, 2, 3, ..., 17  
 $X = \# \text{ dots on face of fair 17-sided die}$   
 $E(X) = \frac{n+1}{2}$

RV w/ 2 or more equally likely outcomes.

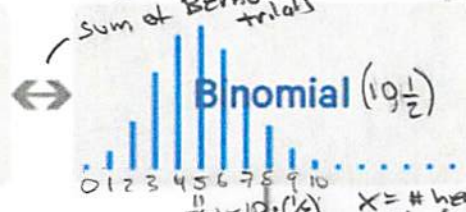
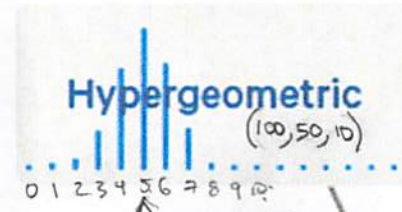
0, 1  
 $X = \# \text{ head you get flipping fair coin once,}$



Sum of Bernoulli trials  
 $E(X) = \frac{18}{2}$

Sum of Bernoulli trials  
 $E(X) = \frac{1}{2}$

Sum of Bernoulli trials

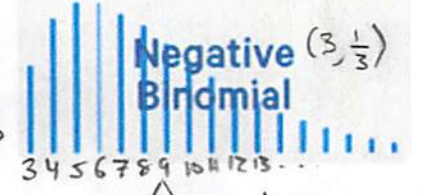


Sum of iid geom (p)

$E(X) = 10 \cdot (\frac{50}{100})$   
 $X = \# \text{ good elements from a size 10 sample drawn (w/o replacement) from a box w/ 100 tickets of which 50 are good.}$   
 $X = 0, 1, \dots, 10$

$E(X) = 10 \cdot (\frac{1}{2})$   
 $X = \# \text{ heads in 10 coin tosses of fair coin.}$   
 $X = 0, 1, 2, \dots, 10$

$E(X) = \frac{1}{\frac{1}{2}} = 2$   
 $X = 1, 2, 3, \dots$   
 $X = \# \text{ coin tosses of prob } P = \frac{1}{2} \text{ coin until first head,}$



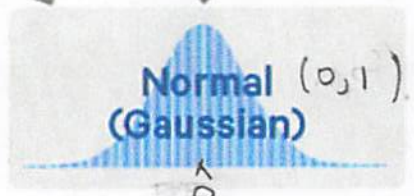
$E(X) = 4$   
 $X = \# \text{ cars through toll booth in 1 min.}$   
 $X = 0, 1, 2, \dots$

$E(X) = 3 \cdot \frac{1}{\frac{1}{3}} = 9$   
 $X = 0, 1, 2, \dots$   
 $X = \# \text{ coin tosses of } P = \frac{1}{3} \text{ coin until 3rd head.}$

CLT

-CLT

CLT



$X = \text{wt of metal in steel 134 in steel units.}$   
 $-\infty < X < \infty$