Chap 4 Continuous Distributions

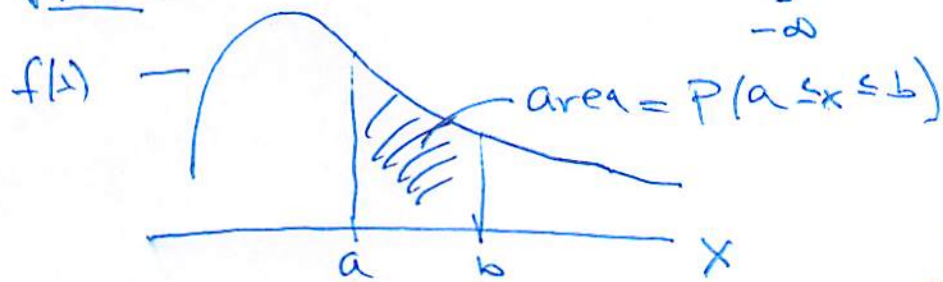
Sec 4.1

Prob densities

let X be continuous RV

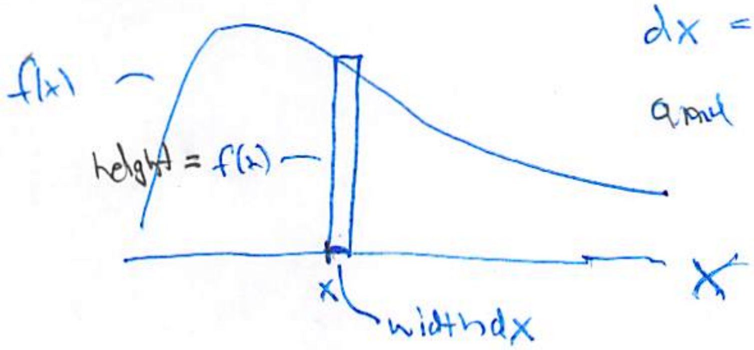
The prob dist of X is described by a probability density function $f(x)$ $\left(\begin{array}{l} f(x) \geq 0 \text{ for all } x \in X \\ \text{and } \int_{-\infty}^{\infty} f(x)dx = 1 \end{array} \right)$.

Picture



For all a, b $P(a \leq x \leq b) = \int_a^b f(x)dx$.

idea



dx = tiny interval around x and also the length of the interval.

$$P(X \in dx) \approx \frac{\overset{\text{area of strip}}{f(x)dX}}{\text{area under curve} = 1} = f(x)dx$$

Note $f(x)$ is not a probability

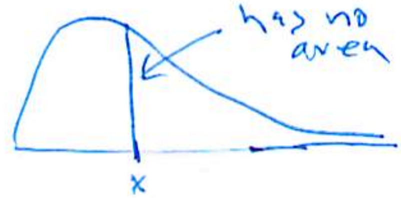
$f(x)dx$ is a prob.

$$f(x) \approx \frac{P(X \in dx)}{dx}$$

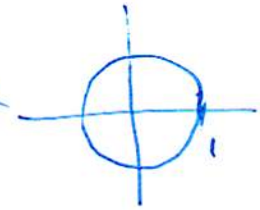
units of f ? — Prob per unit length.

Hence "prob density".

$P(X = x) = 0$

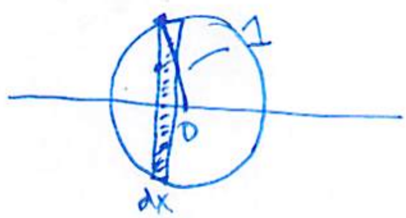


ex Throw down a point uniformly on a unit disk



X: x coordinate of the point.

Pos values of X? — $[-1, 1]$

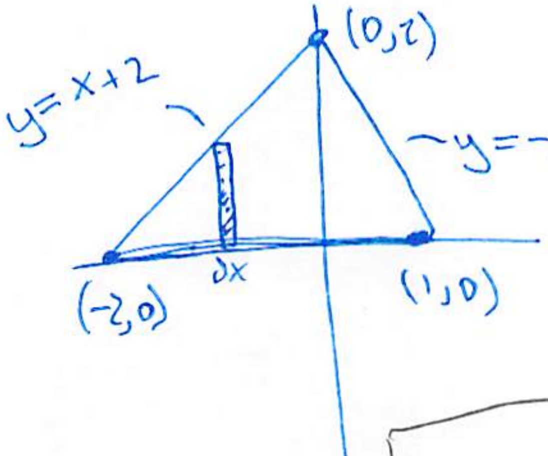


I-clicker question (see next page)

ex 4.1.12 b

Consider a point picked uniformly at random from the area inside the shape below.

Find the density function of the x coordinate.



total area = 3

$$P(X \in dx) = \begin{cases} \frac{(x+2)dx}{3} & \text{if } -2 \le x < 0 \\ \frac{(-2x+2)dx}{3} & \text{if } 0 \le x \le 1 \end{cases}$$

$$\boxed{f(x) = \begin{cases} 0 & \text{else} \\ \frac{x+2}{3} & \text{if } -2 \le x \le 0 \\ \frac{-2x+2}{3} & \text{if } 0 \le x \le 1 \end{cases}}$$

Stat 134

Monday March 5 2018

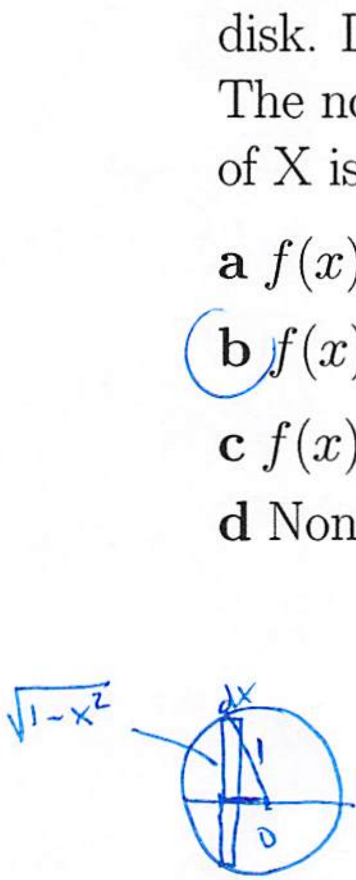
1. Throw down a point uniformly on a unit disk. Let X be the x coordinate of the point. The nonzero part of the probability density of X is :

   a $f(x) = 2\sqrt{1-x^2}$

   b $f(x) = \frac{2\sqrt{1-x^2}}{\pi}$

   c $f(x) = \frac{\sqrt{1-x^2}}{2\pi}$

   d None of the above



$$P(X \in dx) = \frac{\text{area of strip}}{\text{area of circle}} = \underbrace{\frac{2\sqrt{1-x^2}}{\pi}}_{f(x)}\,dx$$

$$\boxed{f(x) = \begin{cases} \frac{2\sqrt{1-x^2}}{\pi} & \text{for } -1 \le x \le 1 \\ 0 & \text{else} \end{cases}}$$

Expectation and variance

For discrete, $E(g(X)) = \sum_{\text{all } x} g(x) P(X=x)$ if $E(|g(X)|) < \infty$

For continuous.

$$E(g(X)) = \int_{-\infty}^{\infty} g(x) P(X \in dx) = \int_{-\infty}^{\infty} g(x) f(x)\,dx$$

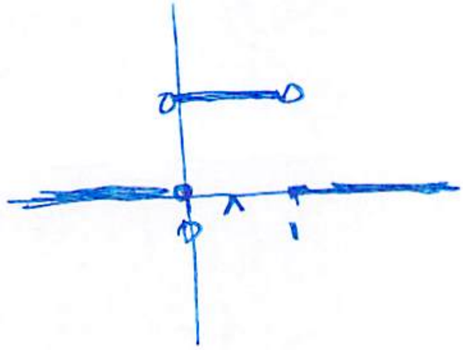
if $E(|g(X)|) < \infty$.

$$\boxed{E(X) = \int_{-\infty}^{\infty} x f(x)\,dx}$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$\boxed{Var(X) = E(X^2) - (E(X))^2}$$

ex U : unif $(0,1)$ ← called std uniform



$$f(u) = \begin{cases} 1 & \text{if } 0<u<1 \\ 0 & \text{else} \end{cases}$$

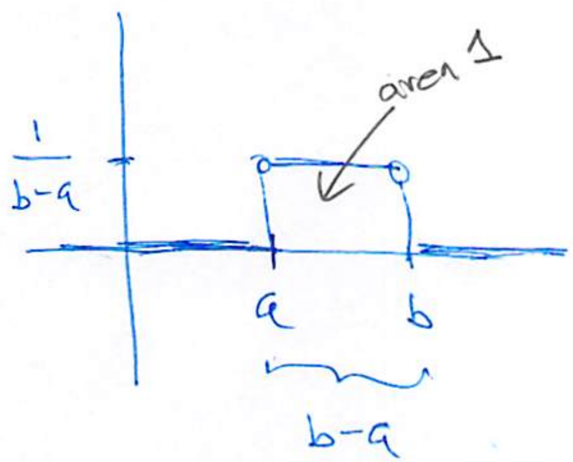
$$E(U) = \int_0^1 u \cdot 1\,du = \frac{u^2}{2}\Big|_0^1 = \boxed{\frac{1}{2}}$$

$$E(U^2) = \int_0^1 u^2 \cdot 1\,du = \frac{u^3}{3}\Big|_0^1 = \frac{1}{3}$$

$$Var(U) = \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \frac{1}{12}$$

lets generalize,

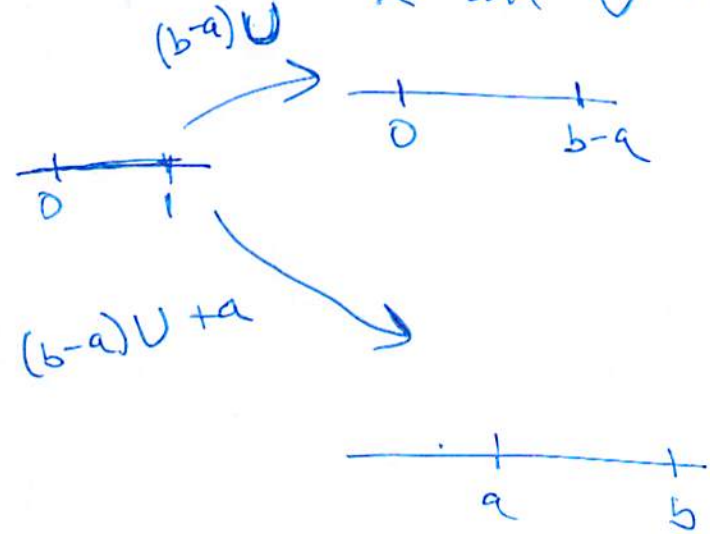
$X : \text{unif}(a, b)$



so area under $f(x) = 1$

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{else} \end{cases}$$

<u>Trick</u> Find relationship between X and U:



$X = (b-a)U + a$

$X = (b-a)U + a$

$E(X) = (b-a)\underbrace{E(U)}_{=\frac{1}{2}} + a = \boxed{\frac{b+a}{2}}$

$Var(X) = (b-a)^2 \underbrace{Var(U)}_{=\frac{1}{12}} = \boxed{\frac{(b-a)^2}{12}}$