Answer T/F

① The top picture
is probability and
bottom picture is
statistics  F

"inverse" probability

- Statistic ↕

Given the information in your hand, what is in the pail?

?

- Prob

Given the information in the pail, what is in your hand?

?

② Section starts today  T

③ lectures will be recorded and can be found on
Stat 134.org  T

④ You should download the prelecture notes on
bCourses / pages before class.  T

⑤ I am optimistic I will be able to take people
off the WL.  T

⑥ My OH is in SLC after class  — T when meet
in person.

11 - 11:30 on
zoom
start 2
weeks.

Today

① Sections 1.1 - 1.3

① <u>Sec 1.1</u>  <u>Equally likely outcomes</u>

We call the set of all outcomes of an experiment
$\Omega$, the <u>outcome space</u>, or the <u>Sample space</u>.

let $A \subseteq \Omega$

$$P(A) = \frac{\#A}{\#\Omega}$$

Deck of cards: 4 suits  H, C, D, S
               13 ranks  Ace, 2-10, J, Q, K
               _____
               52 Cards

<u>ex</u>   ___#se p10
   Suppose a deck of cards is shuffled and the top
2 cards are dealt. What is the chance you get
at least one ace among the 2 cards

$$1 - P(A^c) \quad = \quad 1 - \frac{48 \cdot 47}{52 \; 51} = \boxed{.149}$$

$\underset{\substack{\text{no ace} \\ \text{in 2 cards}}}{\phantom{c}}$

**Ex** Two draws are made at random <mark>with replacement</mark> from the box

$$\lfloor \boxed{1}\boxed{2}\boxed{3}\boxed{4}\boxed{5}\boxed{6}\boxed{7}\boxed{8}\boxed{9}\boxed{10} \rfloor$$

Find the chance the 2nd number is bigger than twice the first.

$\Omega$ = all pairs of numbers $\left( \#\Omega = 10 \cdot 10 = 100 \right)$

$A = \begin{cases} (1, >2) \text{ --- } 8 \\ (2, >4) \text{ --- } 6 \\ (3, >6) \text{ --- } 4 \\ (4, >8) \text{ --- } 2 \end{cases}$

$\Rightarrow \#A = 20$

$P(A) = \dfrac{\#A}{\#\Omega} = \dfrac{20}{100} = \boxed{.2}$

<u>Sec 1.2  Interpretations</u>
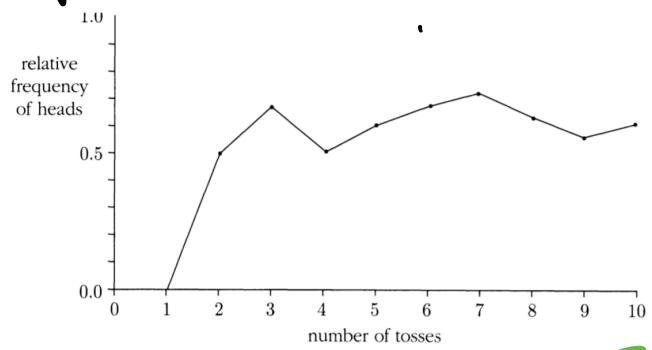
Probability has 2 interpretations

a) ==frequency interpretation.==

es what is the probability a particular coin
lands heads.

There is a correct answer. The answer isn't a
random variable.

Make an experiment, Law of averages.



b) ==Bayesian interpretation.==  <span style="color:green">will discuss in section 1.5</span>

es what is the probability a particular coin
lands heads.

There is no correct answer. The probability is a
random variable

Your opinion may change over time as you
acquire new data. this will change the value
of your probability.

<u>Sec 1.3</u> <u>Distributions</u>

To define probability we start with an <u>outcome</u>
<u>space</u>, $\Omega$, and assign to each element a nonnegative
number and require that all numbers add up to 1.

<u>Axioms</u>

<u>Venn Diagram</u>
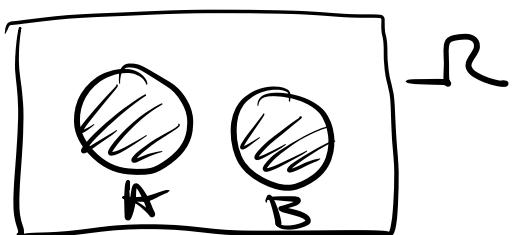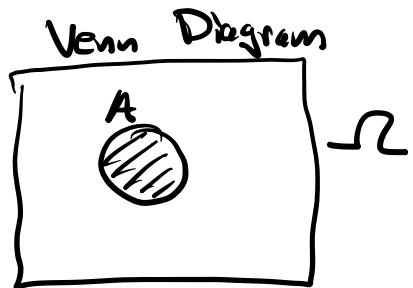
1) $P(A) \geq 0$ for all $A \subseteq \Omega$



2) $P(\Omega) = 1$

3) If A and B are mutually exclusive
    sets then $P(A \cup B) = P(A) + P(B)$

(addition rule)
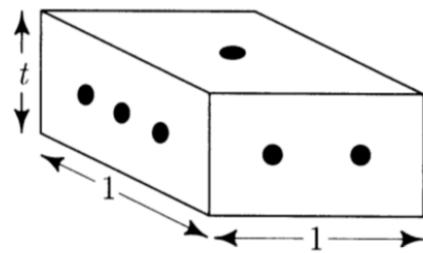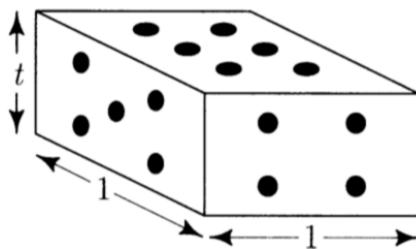
$\underline{\underline{ex}}$

**Example 3.**  **Shapes.**

P24

A *shape* is a 6-sided die with faces cut as shown in the following diagram:



Suppose the thickness of the die, $t$, is such that the chance of landing flat (1 or 6) is $2/3$.

Find the probability distribution of the shape. Draw a histogram.

$\frac{1}{3} = 4 \cdot x$

← chance of getting a non-flat side

↑
chance get
dont get 1 or 6

$\Rightarrow \boxed{x = \frac{1}{12}}$

$P(1) = \frac{1}{3}$

$\left. \begin{array}{l} P(2) \\ P(3) \\ P(4) \\ P(5) \end{array} \right\} = \frac{1}{12}$

$P(6) = \frac{1}{3}$

$(\frac{1}{3})$  $(\frac{1}{3})$

$(\frac{1}{12})$

1  2  3  4  5  6
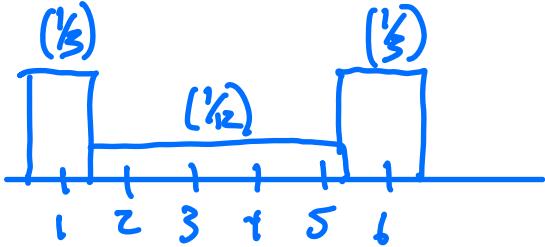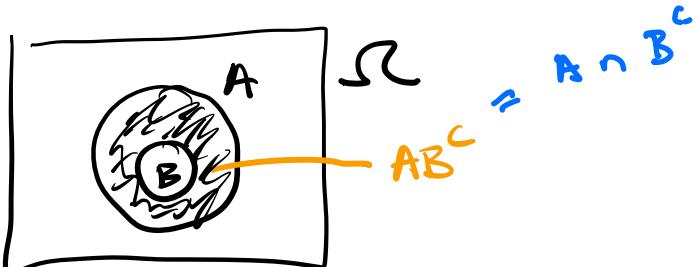
# Difference rule

Suppose $B \subseteq A$



$\Omega$

$AB^c = A \cap B^c$

Prove a formula for $P(AB^c)$ in terms of $P(A)$ and $P(B)$.

$P(A) - P(B)$

$A = B \cup AB^c$ disjoint union

$P(A) = P(B) + P(AB^c)$ by add$^n$ rule.
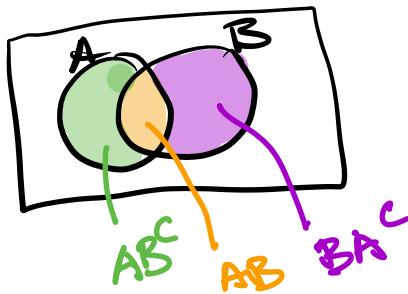
$\Rightarrow P(AB^c) = P(A) - P(B)$ $\square$

# Inclusion exclusion rule

$$P(A \cup B) = P(A) + P(B) - P(AB)$$
$$\uparrow A \cap B$$

Proof/



$A \cup B = AB^c \cup AB \cup BA^c$   disjoint union

$P(A \cup B) = \underbrace{P(AB^c)}_{=} + \underbrace{P(AB)}_{=} + \underbrace{P(BA^c)}_{=}$   by add$^n$ rule

$\phantom{P(A \cup B) = }P(A) - P(AB) \qquad P(AB) \qquad P(B) - P(AB)$

$$= \boxed{P(A) + P(B) - P(AB)}$$