Stat 134 lec 12

Warmup 10:00–10:10

$X \sim Pois\left(\frac{1}{3}\right)$

Find $E(X!)$

$X \sim Pois(\mu)$

$P(X=k) = \dfrac{e^{-\mu}\mu^k}{k!}$

$k = 0, 1, 2, \ldots$

$E(g(x)) = \sum_{\text{all } x} g(x) P(X=x)$

$\dfrac{1}{1-1/3} = 3/2$

$E(X!) = \sum_{x=0}^{\infty} x!\, P(X=x) = \sum_{x=0}^{\infty} \cancel{x!} \dfrac{e^{-1/3}(1/3)^x}{\cancel{x!}} = e^{-1/3}\left(1 + 1/3 + (1/3)^2 + \cdots\right)$

$= \boxed{\frac{3}{2} \cdot e^{-1/3}}$

last time

Sec 3.3 SD(X) is the average deviation from the mean

i.e $SD = \sigma = \sqrt{E((X-\mu)^2)}$

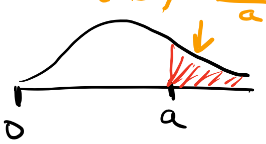
$Var = \sigma^2 = E((X-\mu)^2)$

often write $E(X-\mu)^2$

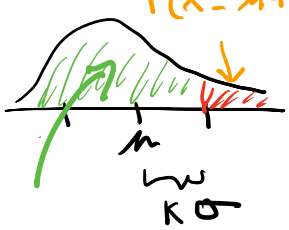
Tail bounds

Markov's inequality

$P(X \geq a) \leq \frac{\mu}{a}$



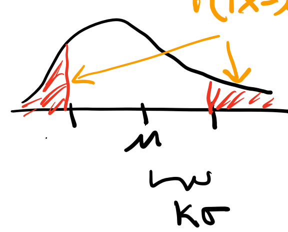
$P(X \geq \mu + k\sigma) \leq \frac{\mu}{\mu + k\sigma}$



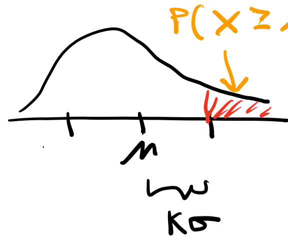
$P(X < \mu + k\sigma) \geq 1 - \frac{\mu}{\mu + k\sigma}$

Chebyshev's inequality

$P(|X-\mu| \geq k\sigma) \leq \frac{1}{k^2}$



$P(X \geq \mu + k\sigma) \leq \frac{1}{k^2}$



(3 pts) Suppose that each year, Berkeley admits 15,000 students on average, with an SD of 5,000 students. Assuming that the application pool is roughly the same across years, find the smallest upper bound you can on the probability that Berkeley will admit at least 22,500 students in 2019. (Hint: you should be comparing two possible bounds.)

let X = # students Cal admits in 2019

M $P(X \geq 22{,}500) \leq \frac{15000}{22{,}500} = 2/3$

C $P(X \geq 22{,}500) \leq \frac{1}{(1.5)^2} = \boxed{4/9}$ ← smaller upper bound

$22{,}500 = 15000 + 1.5\,(5000)$

$k = 1.5$

Today Sec 3.3

1. another formula for variance
2. Properties of variance

1

Sec 3.3 Another formula for $Var(X)$.

Recall $E(cX) = cE(X)$

so $E(E(X)X) = E(X)E(X)$

$$Var(X) = E\left((X - E(X))^2\right)$$

FOIL

$$= E\left(X^2 - 2E(X)X + (E(X))^2\right)$$

$E(cX) = cE(X)$

$$= E(X^2) \underbrace{- 2E(X)E(X) + (E(X))^2}_{= -E(X)^2}$$

$$\Rightarrow \boxed{Var(X) = E(X^2) - E(X)^2}$$

$$\text{(or)} \quad \boxed{E(X^2) = Var(X) + E(X)^2}$$

$(E(X))^2$

ex let $X = \begin{cases} 1 & \text{with prob } p \\ 0 & \text{with prob } q \end{cases}$

$$E(X^2) = \sum_{\text{all } x} x^2 P(X=x) = 1^2 \cdot p + 0 \cdot q = \boxed{p}$$

$Var(X)$? $\quad p - p^2 = p(1-p) = \boxed{pq}$

eg Let X be a nonnegative RV such that

$E(X) = 100 = \operatorname{var}(X)$

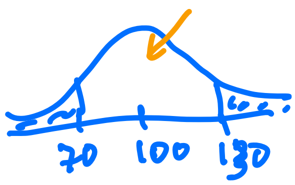
a) Can you find $E(X^2)$ exactly? If not what can you say.

$$E(X^2) = \operatorname{var}(X) + (E(X))^2$$
$$= 100 + 10{,}000 = \boxed{10{,}100}$$

b) Can you find $P(70^2 < X^2 < 130^2)$ exactly? If not what can you say?

$$P(70^2 < X^2 < 130^2) = P(70 < X < 130) \geq 1 - \frac{1}{k^2} = \boxed{\frac{8}{9}}$$

since X is nonnegative



$\leq \frac{1}{9}$

$k=3$

Note

$$P(70^2 < X^2 < 130^2) = P(-130 < X < -70) + P(70 < X < 130)$$

zero since X is nonnegative

M: $P(X \geq \sqrt{40}) \leq \frac{E(X)}{\sqrt{40}}$

$= 3/\sqrt{40}$

Stat 134

1. X is nonnegative random variable with $E(X) = 3$ and $SD(X) = 2$. True, False or Maybe:

$$P(X^2 \geq 40) \leq \frac{1}{3}$$

a True

b False

c Maybe

C: $P(X^2 \geq 40) = P(X \geq \sqrt{40}) \leq \frac{1}{K^2} = \frac{1}{\left(\frac{\sqrt{40}-3}{2}\right)^2} = 0.36$

$\sqrt{40} = \mu + K\sigma$, $\mu = 3$, $\sigma = 2$

$K = \frac{\sqrt{40}-3}{2}$

Note we don't know $SD(X^2)$

M: $E(X^2) = \underbrace{var(X)}_{=4} + \underbrace{E(X)^2}_{=9} = 13$

$P(X^2 \geq 40) \leq \frac{E(X^2)}{40} = \frac{13}{40} < \frac{1}{3}$

Note Markov provides another inequality:

$P(X^2 \geq 40) = P(X \geq \sqrt{40}) \leq 3/\sqrt{40}$ but this gives a maybe.