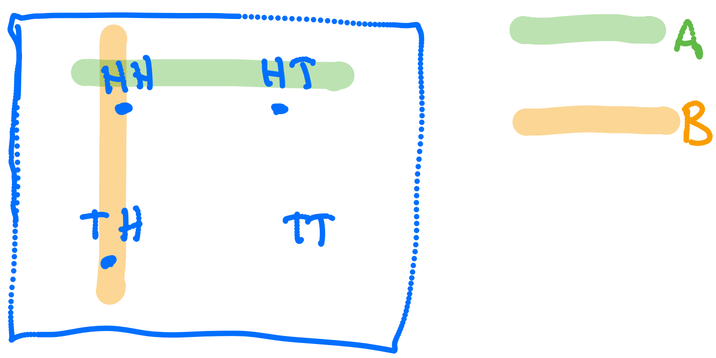
Stat 134 lec 3

warmup 10:00-10:10

Flip two coins. Find two independent events of the outcome space.
Show the events in a Venn diagram.

A = heads on 1st toss
B = heads on 2nd toss



Note that if A, B are indep and nonempty then they must have a nonempty intersection.

mutually exclusive $\Rightarrow$ dependent

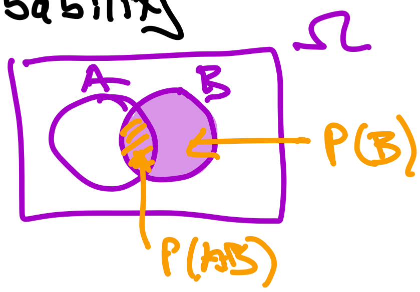
independent $\Rightarrow$ not mutually exclusive

last time

Sec 1.4 Conditional Probablity

$A|B$ "A given B"

New sample space is B,



$$P(A|B) = \frac{P(AB)}{P(B)}$$ Baye's Rule.

Or $P(AB) = P(A|B)P(B)$ multiplication Rule

$= P(A)P(B)$ (if A and B are independent).

Inclusion Exclusion Rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

ex

Two separate decks of cards are shuffled. What is the chance that the top card of the first deck is the **king** of spades **or** the bottom card of the second deck is the **king** of spades

a $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{52}$ $= P(AB) = P(A)P(B)$

b $\frac{1}{52} + \frac{1}{51}$

c $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{51}$

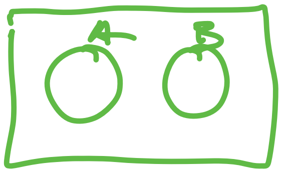
d none of the above

Today

1. Sec 1.4 Mutually Exclusive versus Independent
2. Sec 1.5 Baye's Rule

① <u>Sec 1.4</u> Mutually Exclusive (ME) versus Independent

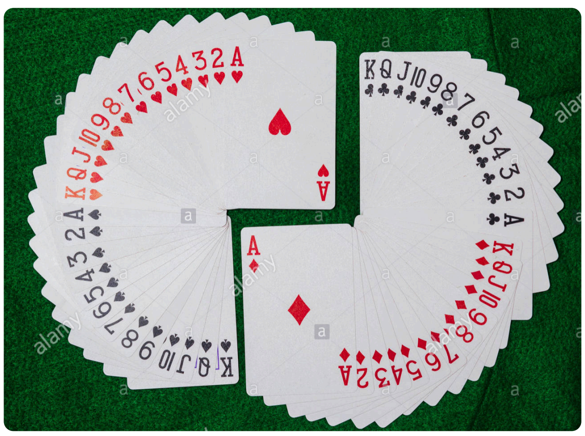
ME: $P(AB) = 0$



Ind: $P(A|B) = P(A)$

eg Consider different kinds of cards

Is red and Heart ~~ME~~, Ind ?



$P(R|H) \neq P(R) \Rightarrow$ dep

$P(R|H) = 1$, $P(R) = 1/2$

$P(H|R) \neq P(H) \Rightarrow$ dep.

$P(H|R) = 1/2$, $P(H) = 1/4$

eg Is red and Spade ME, Ind ?

$P(R|S) \neq P(R) \Rightarrow$ dep

$P(R|S) = 0$, $P(R) = 1/2$

If A, B are nonempty sets

A, B ME is A, B Dependent ?

$AB = \phi$

$A, B \neq \phi$

$$P(A|B) = \frac{P(AB)^{=0}}{P(B)_{\neq 0}} = 0$$

$P(A) \neq 0$ $\Rightarrow$ $\boxed{\text{dep}}$

Sec 1.5 Baye's rule

ex A factory produces 2 models of cell phones.

Given $P(1) = .3$ — Prior prob

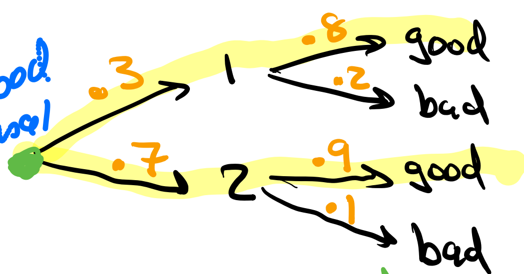
$P(\text{good} \mid 1) = .8$

$P(\text{good} \mid 2) = .9$

forward conditional

or likelihood conditional

$P(A \text{ and } B)$
$P(AB)$
$P(A,B)$
$P(A \cap B)$
$\}$ all the same



Find $P(1, \text{good})$ (and) $= P(1)P(\text{good} \mid 1) = (.3)(.8) = \boxed{.24}$

$P(\text{good}) = P(1,\text{good}) + P(2,\text{good})$

$= (.3)(.8) + (.7)(.9) = \boxed{.87}$

$P(1 \mid \text{good}) = \dfrac{P(1,\text{good})}{P(\text{good})} = \dfrac{.24}{.87} = \boxed{.28}$

need Bayes rule

"Backword conditional"

or posterior conditional

$P(1,\text{good}) = P(\text{good},1) = P(\text{good})P(1 \mid \text{good})$

$= P(1)P(\text{good} \mid 1)$

Proportional

Note $P(1 \mid \text{good}) = \dfrac{1}{P(\text{good})} \cdot P(1)\, P(\text{good} \mid 1)$

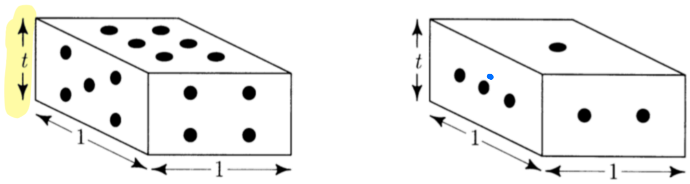
posterior prior likelihood

constant.

ex

Shapes.

A *shape* is a 6-sided die with faces cut as shown in the following diagram:



A box contains 3 shaped die (see pic above), D_1, D_2, D_3, with probability $1/3$, $1/2$, $2/3$ respectively of landing flat (with 1 or 6 on top).

Note: the numbers $1/3, 1/2, 2/3$ dont add up to 1 because they are the chance of landing flat for 3 different die.

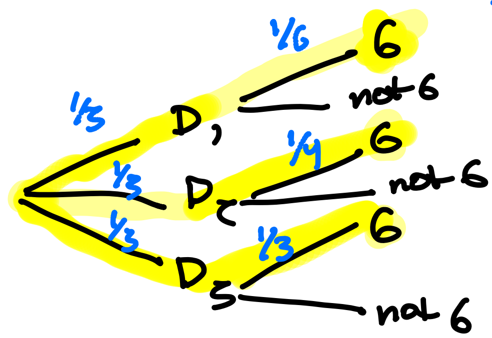
a) what is $P(\text{get } 6 \mid D_1) = \frac{1}{2} \cdot \frac{1}{3} = \boxed{1/6}$ (likelihood)

b) what is $P(\text{get } 6, D_1) = P(\text{get } 6 \mid D_1) \cdot P(D_1)$
$= 1/6 \cdot 1/3 = \boxed{1/18}$

c) what is $P(\text{get } 6)$ ~

$\frac{1}{3} \cdot 1/6 + \frac{1}{3} \cdot 1/4 + 1/3 \cdot 1/3$

$= \boxed{1/4}$



d) Find posterior $P(D_1 \mid 6) = \frac{P(D_1, 6)}{P(6)} = \frac{1/18}{1/4} = \boxed{2/9}$

Suppose you draw a number from a bag, with equal probabilities across the choices {1, 2, 3}.

Once you draw a number, you toss a coin until you get that many number of heads followed by a tails—so if you draw a 3, you keep tossing until you encounter the sequence {Heads, Heads, Heads, Tails}.

What is the probability of tossing a coin seven times given that you draw the number 2?

$P(7 \mid \text{draw } 2)$ ← forward conditional (don't need Bayes's rule)

$$\underbrace{* * * *}_{\text{anything but } HHT* \text{ or } *HHT} HHT$$

16 poss minus 4 = 12 possibilities

$$\frac{12}{2^7} = \boxed{12\left(\tfrac{1}{2}\right)^7}$$