Stat 134 lec 8

warmup 10:00–10:10

Find the probability that a poker hand has two 2 of a kind

ex K, K, Q, Q, 7

$$\frac{\binom{13}{2}\binom{11}{1}\overset{K}{\binom{4}{2}}\overset{Q}{\binom{4}{2}}\overset{7}{\binom{4}{1}}}{\binom{52}{5}}$$

Find the probability of being dealt a three of a kind poker hand (ranks aaabc where $a \neq b \neq c$)

$$\left.\begin{aligned} &\frac{\binom{13}{2}\binom{4}{1}\binom{4}{1}\binom{11}{1}\binom{4}{3}}{\binom{52}{5}} \\ = &\frac{\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}\binom{4}{1}}{\binom{52}{5}} \end{aligned}\right\} \text{both are correct}$$

Last time

independent trials (draw w/ replacement)
- binomial distribution — 2 outcome trial
- multinomial distribution — k outcome trial

dependent trials (draw w/o replacement)
- hypergeometric distribution — 2 outcom trial
- multivariate hypergeometric distribution — k outcome trial

sec 2.5 hypergeometric distribution

abbrev. $HG(n, N, G)$

Parameters:
N = population size
G = # Good in population
n = sample size.

Suppose a population of size N contains G good and B bad elements $(N=G+B)$. A sample, size n, with g good and b bad elements $(n=g+b)$ is chosen at random without replacement.

$$P(g \text{ good}) = \frac{\binom{G}{g}\binom{B}{b}}{\binom{N}{n}}$$

today

① sec 2.5 Binomial approx to hypergeometric.

② sec 3.1 — random variables (RV)
— joint distribution of 2 RVs and independence

① Sec 2.5 Binomial approx to hypergeometric.

Binomial — independent trials
Hypergeometric — dependent trials.

ex 100 person class with a grade distribution :

A grade : 70 students
B grade : 30 students.

Sample 5 students at random w/o replacement (SRS).

Find $P(3A's, 2Bs)$

exact
hypergeometric
$$= \frac{\binom{70}{3}\binom{30}{2}}{\binom{100}{5}} = \binom{5}{3}\frac{70}{100}\frac{69}{99}\frac{68}{98}\frac{30}{97}\frac{29}{96} = \boxed{.316}$$

approx
binomial
$$= \binom{5}{3}(.7)^3(.3)^2 = \boxed{.309}$$

when N is large relative to n, $HG(5,100,70) \approx Bin(5,.7)$

why?

$$HG(n,N,G) \approx Bin\left(n, \frac{G}{N}\right)$$

Summary of approximations

$HG(n,N,G)$

↓ approx by binomial
N large, n small
$p = \frac{G}{N}$

binomial (n,p)

↙ approx by Poisson
$p \to 0$, $n \to \infty$, $np \to \mu$

Poisson(μ)

↘ approx by normal
n large
$\mu = np$, $\sigma = \sqrt{npq}$
$0 < \mu \pm 3\sigma < n$
use continuity correction

Normal(μ, σ^2)

② Sec 3.1 Intro to Random Variables (RV)

A RV, X, is the outcome of an experiment.

What distribution is the following RV?

X = The number of aces in 5 cards drawn from a standard deck?

$HG(5,52,4)$

ex flip a prob p coin 2 times

$X = \#$ heads

we write $X \sim Bin(2,p)$

More precisely,

$$X : \overset{\text{outcome space}}{\Omega} \longrightarrow \mathbb{R} \quad \text{is a function}$$

$$\begin{aligned} HH &\longmapsto 2 \\ HT &\longmapsto 1 \\ TH &\longmapsto 1 \\ TT &\longmapsto 0 \end{aligned}$$

so $X=1$ means $\{HT, TH\} \subseteq \Omega$

$X=1$ is an event

$P(X=1) = \binom{2}{1} p^1 (1-p)^2$ binomial formula

Joint Distribution

Let (X,Y) be the joint outcome of 2 RVs X, Y.

ex

X: one draw from $\boxed{\boxed{1}\ \boxed{2}\ \boxed{2}\ \boxed{3}}$

Given $X=x$, Y = number of heads in x coin tosses.

$$P(X=x, Y=y) = P(Y=y \mid X=x) \cdot P(X=x)$$

$$P(X=1, Y=1) = \underset{\overset{''}{1/2}}{P(Y=1 \mid X=1)} \cdot \underset{\overset{''}{1/4}}{P(X=1)} = \boxed{1/8}$$

What the range of values of X? — 1, 2, 3

Y? — 0, 1, 2, 3

Find,

$P(1,0) = \underset{\overset{''}{1/2}}{P(Y=0 \mid X=1)}\, \underset{\overset{''}{\frac{1}{4}}}{P(X=1)} = 1/8$

$P(1,1) = \frac{1}{2} \cdot \frac{1}{4} = \boxed{1/8}$

$P(2,0) = \frac{1}{4} \cdot \frac{1}{2} = 1/8$

$P(2,1) = 1/2 \cdot 1/2 = 1/4$

$P(2,2) = \frac{1}{4} \cdot \frac{1}{2} = \boxed{1/8}$

$P(3,0) = 1/8 \cdot 1/4 = 1/32$

$P(3,1) = \frac{3}{8} \cdot \frac{1}{4} = \boxed{\frac{3}{32}}$

$P(3,2) = \frac{3}{8} \cdot \frac{1}{4} \quad \boxed{\frac{3}{32}}$

$P(3,3) = \frac{1}{8} \cdot \frac{1}{4} \quad \boxed{\frac{1}{32}}$

	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	
3	0	0	$\frac{1}{32}$	$\frac{1}{32}$
2	0	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{7}{32}$
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{32}$	$\frac{15}{32}$
0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{9}{32}$
Y X	1	2	3	

marginal prob of X

$P(x) = \sum_{y \in Y} P(x,y)$

marginal prob of Y

$P(y) = \sum_{x \in X} P(x,y)$

Is X, Y dependent? — dependent

eg

$$\left.\begin{array}{l} P(Y=0 \mid X=1) = \frac{1}{2} \\ P(Y=0) = 9/32 \end{array}\right\} \Rightarrow X, Y \text{ dep}$$

Def^n Two RVs are **independent** if

$$P(Y=y \mid X=x) = P(Y=y) \quad \text{for all } \begin{matrix} x \in X \\ y \in Y \end{matrix}$$

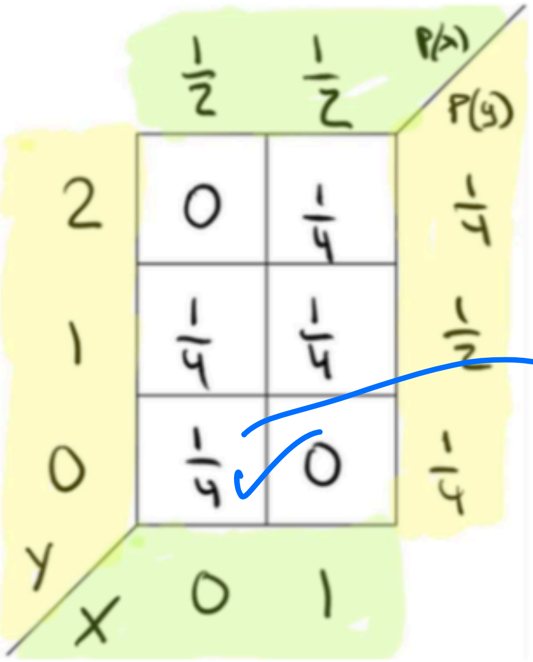
By the multiplication rule,

if X, Y are indep,

$$P(X=x, Y=y) = \underset{\substack{\parallel \\ P(Y=y)}}{P(Y=y \mid X=x)}\, P(X=x)$$

so $$\boxed{P(X=x, Y=y) = P(X=x)P(Y=y)}.$$

A fair coin is tossed twice.
Let X = # heads on the first toss.
Let Y = # heads on the first 2 tosses.



Y \ X	0	1	P(y)
2	0	$\frac{1}{4}$	$\frac{1}{4}$
1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$
0	$\frac{1}{4}$	0	$\frac{1}{4}$
P(x)	$\frac{1}{2}$	$\frac{1}{2}$	

$P(X=0, Y=0)$

$\underset{1/2}{\underset{\shortparallel}{P(Y=0|X=0)}}\;\underset{1/2}{\underset{\shortparallel}{P(X=0)}} = 1/4$

a The table above is correct ✓

b $Y \sim Bin(2, \frac{1}{2})$ ✓

[✓ **c**] More than one of the above

d None of the above

Check every entry in table