Stat 134 lec 18

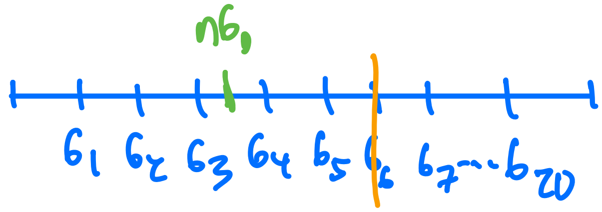
Warmup 11:00 – 11:10

An urn contains 90 marbles, of which there are 20 greens, 20 blacks and 50 red marbles. Tom draws marble without replacement until the 6^{th} green marble. Let $X = \#$ of marbles drawn. Example: **GGG**BRB**GG**BR**G** with $x = 11$. Find $\mathbb{E}[X]$ and Var(X)

$$X = I_1 + \cdots + I_{70} + 6$$

$$p_1 = 6/21$$

$$I_2 = \begin{cases} 1 & \text{if } 2^{nd} \text{ nongreen before } 6^{th} \text{ green} \\ 0 & \text{else} \end{cases}$$



$$\boxed{E(X) = 70\left(6/21\right) + 6}$$

$$p_{12} = \frac{6}{21} \cdot \frac{7}{22}$$

$$I_{12} = \begin{cases} 1 & \text{if } 1^{st} \text{ and } 2^{nd} \text{ nongreen before } 6^{th} \text{ green} \\ 0 & \text{else} \end{cases}$$

let $Y = I_1 + \cdots + I_{70}$

$$Var(Y) = Var(X)$$

$$E(Y^2) = 70p_1 + 70 \cdot 69 p_{12}$$

$$E(Y) = E(X) - 6 = 70\left(6/21\right)$$

$$Var(X) = Var(Y) = E(Y^2) - [E(Y)]^2$$

$$= \boxed{70p_1 + 70 \cdot 69 p_{12} - \left(70\left(6/21\right)\right)^2}$$

Today MT1 Review

Joshua Parker

6:22pm

I would like to go over these problems from the textbook.

3.2 # 13

3.3# 8, 15 (a and b)

Reply

13. Suppose a fair die is rolled ten times. Find numerical values for the expectations of each of the following random variables:

a) the sum of the numbers in the ten rolls;

b) the sum of the largest two numbers in the first three rolls;

c) the maximum number in the first five rolls;

d) the number of multiples of three in the ten rolls;

e) the number of faces which fail to appear in the ten rolls;

f) the number of different faces that appear in the ten rolls;

b) X = sum of largest 2 in first 3 rolls

$= X_1 + X_2 + X_3 - \min(X_1, X_2, X_3)$

recall tail sum formula: $E(Y) = \sum_{k=1}^{\infty} P(Y \geq k)$

$P(\min(X_1, X_2, X_3) \geq k) = P(X_1 \geq k, X_2 \geq k, X_3 \geq k)$

$= P(X_1 \geq k)^3$

$$\Rightarrow E(\min(X_1, X_2, X_3)) = \sum_{k=1}^{\infty} (P(X_1 \geq k))^3$$

$$= 1^3 + \left(\frac{5}{6}\right)^3 + \left(\frac{4}{6}\right)^3 + \left(\frac{3}{6}\right)^3 + \left(\frac{2}{6}\right)^3 + \left(\frac{1}{6}\right)^3$$

$$E(X_1) = \frac{1}{6}(1+2+3+4+5+6) = \frac{7}{2}$$

$$\Rightarrow E(X) = \boxed{3 \cdot \frac{7}{2} - 1^3 + \left(\frac{5}{6}\right)^3 + \left(\frac{4}{6}\right)^3 + \left(\frac{3}{6}\right)^3 + \left(\frac{2}{6}\right)^3 + \left(\frac{1}{6}\right)^3}$$

c) $P(\max(X_1, .., X_5) < k) = P(X_1 < k)^5$

$$\Rightarrow P(\max(X_1, .., X_5) \geq k) = 1 - P(X_1 < k)^5$$

$$\Rightarrow E(\max(X_1, .., X_5)) = \sum_{k=1}^{6} 1 - P(X_1 < k)^5$$

$$= 6 - \left[0^5 + \left(\frac{1}{6}\right)^5 + \left(\frac{2}{6}\right)^5 + \dots + \left(\frac{5}{6}\right)^5\right]$$

$$= \boxed{5.43}$$

8. Let A_1, A_2, and A_3 be events with probabilities $\frac{1}{5}$, $\frac{1}{4}$, and $\frac{1}{3}$, respectively. Let N be the number of these events that occur.

a) Write down a formula for N in terms of indicators.

b) Find $E(N)$.

In each of the following cases, calculate $Var(N)$:

c) A_1, A_2, A_3 are disjoint;

d) they are independent;

e) $A_1 \subset A_2 \subset A_3$.

c)

$N = I_1 + I_2 + I_3$ where

$p = 1/4$

$I_2 = \begin{cases} 1 & \text{if } A_2 \text{ occurs} \\ 0 & \text{else} \end{cases}$

A_1, A_2, A_3 disjoint means N is an indicator

$p = 1/5 + 1/4 + 1/3 = \frac{47}{60}$

$N = \begin{cases} 1 & \text{if } A_1 \text{ or } A_2 \text{ or } A_3 \text{ occur} \\ 0 & \text{else} \end{cases}$

$Var(N) = \boxed{\frac{47}{60} \cdot \left(1 - \frac{47}{60}\right)}$

$13/60 = 1 - \frac{47}{60}$

d) A_1, A_2, A_3 indep $\Rightarrow I_1, I_2, I_3$ indep

$Var(N) = Var(I_1) + Var(I_2) + Var(I_3)$

$Var(I_1) = \frac{1}{5}\left(\frac{4}{5}\right)$, $Var(I_2) = \left(\frac{1}{4} \cdot \frac{3}{4}\right)$, $Var(I_3) = \left(\frac{1}{3} \cdot \frac{2}{3}\right)$

Denise Robles
9:48pm

I have a question regarding lecture 12, specifically in sec 3.2 of the lecture notes where we do the expectation for a geometric distribution, I'm not exactly sure how we derived it or simplified it.

Ex Suppose $X \sim \text{Geom}(p)$ on $\{1, 2, \cdots\}$ with $p > 2/3$

Find $E(3^X)$.

Picture

$$1 \xrightarrow{y = 3^x} 3^1 \, p$$
$$2 \longrightarrow 3^2 \, qp$$
$$3 \longrightarrow 3^3 \, q^2 p$$
$$\vdots$$

$X \sim \text{Geom}(p)$
\# trials to 1st failure
$P(X=k) = q^{k-1} p$

$$E(3^X) = \sum_{k=1}^{\infty} 3^k P(X=k) = \sum_{k=1}^{\infty} 3^k q^{k-1} p$$
$$= 3p + 3^2 qp + 3^3 q^2 p + \cdots$$
$$= 3p\underbrace{(1 + 3q + (3q)^2 + \cdots)}_{= \frac{1}{1-3q}} \quad \text{if } 3q < 1$$

yes since $p > 2/3$

$$\boxed{E(3^X) = 3p\left(\frac{1}{1-3q}\right)}$$

Answer: we define

$$E(g(X)) = \sum_{x \in X} g(x) P(X = x)$$

In this example $g(X) = 3^X$.

Haijing He

9:53pm

HW 3.4.2 vs 3.4.10

Those two looks the same question, but the solution is totally different.

3.4.2 distribution is 1 + geometric, a very easy solution

However, 3.4.10, the solution is complicated.

I am wanting why 3.4.10 can't be as a 3.4.2, 1 + geometric distribution.

3.4.5 part (d)

3.4.11 part (d)

2. An urn contains 10 red balls and 10 black balls. Balls are drawn out at random with replacement until at least one ball of each color has been drawn out. Let D be the number of draws. Find: a) the distribution of D; b) $E(D)$; c) $SD(D)$.

D = # draws until at least one ball each color

$2, 3, \ldots$

First ball can be anything then have Geom(1/2)

$D = 1 + \text{Geom}(1/2)$

$E(D) = 1 + \frac{1}{1/2} = 3$

$\text{Var}(D) = 2$

prob p prob q

10 ~~**2.**~~ An urn contains ~~10~~ red balls and ~~10~~ black balls. Balls are drawn out at random with replacement until at least one ball of each color has been drawn out. Let D be the number of draws. Find: a) the distribution of D; b) $E(D)$; c) $SD(D)$.

Now no longer $D = 1 + \text{Geom}(1/2)$

If first ball is red then $D = 1 + \text{Geom}(q)$

If first ball is black then $D = 1 + \text{Geom}(p)$

$P(D=k) = q^{k-1}p + p^{k-1}q$ for $k \geq 2$

$E(D) = \sum_{k=2}^{\infty} kP(D=k)$

$var(D) = E(D^2) - (E(D))^2$

11. Suppose that A tosses a coin which lands heads with probability p_A, and B tosses one which lands heads with probability p_B. They toss their coins simultaneously over and over again, in a competition to see who gets the first head. The one to get the first head is the winner, except that a draw results if they get their first heads together. Calculate:

a) $P(\text{A wins})$; b) $P(\text{B wins})$; c) $P(\text{draw})$;

d) the distribution of the number of times A and B must toss.

Let $X \sim Geom(p_A)$ number of tosses until A wins

$Y \sim Geom(p_B)$

X and Y are independent geometrics

$Z = \min(X, Y)$ is number of times A and B must toss.

$Z \sim Geom(1 - q_A q_B)$ (~~see~~ lec 16)