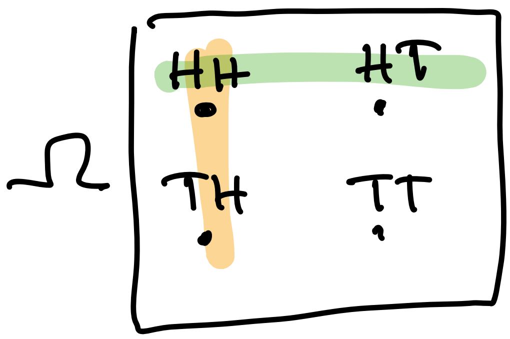
Stat 134 lec 3

warmup

The outcome space of flipping two coins is drawn below.



Find two indep. events and show them in the outcome space.

A = heads on 1^{st} toss

B = heads on 2^{nd} toss

Notice that A, B are independent nonempty sets that have a nonempty intersection,

mutualy exclusive $\Rightarrow$ not independent (dependent) since if you have one you cant have the other.

$\Leftrightarrow$ indendent $\Rightarrow$ not mutually exclusive.

last time

Sec 1.4 Conditional Probability and independence

we saw last time the multiplication rule

$P(AB) = P(A|B)P(B)$ and $P(AB) = P(B|A)P(A)$

$\Rightarrow P(A|B)P(B) = P(B|A)P(A)$

Hence if $P(A|B) = P(A)$

$\underset{\cancel{P(A)}}{\underset{\parallel}{P(A|B)}}P(B) = P(B|A)\cancel{P(A)}$ assume $P(A) \neq 0$,

$\Rightarrow P(B|A) = P(B)$

So to show independence you can show

$\left\{ \begin{array}{l} P(A|B) = P(A) \\ P(B|A) = P(B) \\ P(AB) = P(A)P(B) \end{array} \right.$

$P(AB) = \underset{P(A)}{\underset{\parallel}{P(A|B)}}P(B)$ mult rule

ex

A deck of cards is shuffled. What is the chance that the top card is the **king** of spades **or** the bottom card is the **king** of spades

a $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{52}$

b $\frac{1}{52} + \frac{1}{51}$

c $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{51}$

d none of the above

$P(AB) = 0$

a	The answer is a by the inclusion exclusion rule

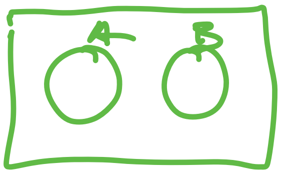
d	Two ways: 1. The two are mutually exclusive events so their intersection is 0 so while taking the union, the individual probabilities add up which is 1/52+1/52 or 1/26 which isn't among the options 2. By the complement rule, the probability is 1 - (51/52)*(50/51) which is 1 - 50/52 which again gives 1/26 which isn't among the options

Today

1. Sec 1.4 Mutually Exclusive versus Independent
2. Sec 1.5 Bayes' Rule

① Sec 1.4 Mutually Exclusive (ME) versus Independent

ME: $P(AB) = 0$



Ind: $P(A|B) = P(A)$

eg Consider different kinds of cards

Is red and Heart ME ✗, Ind ✗ ?

$\underset{1}{P(R|H)} \neq \underset{1/2}{P(R)}$

→ not indep.

eg Is red and Spade ME ✓, Ind ✗ ?

eg Is red and Ace ME ✗, Ind ✓ ? $P(R|Ace) = P(R) = 1/2$

eg Is green and Ace ME ✓, Ind ✓ ?

green $= \emptyset$

$P(\emptyset B) = 0$, $\emptyset B = \emptyset$

$P(\emptyset|B) = \frac{P(\emptyset \cdot B)}{P(B)} = 0$

$P(\emptyset \cdot B) = P(\emptyset)P(B)$

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Suppose A and B are two events with

$$\boxed{P(A) = 0.8 \text{ and } P(A \cup B) = 0.8.}$$

$\Rightarrow B = \emptyset$
$P(B) = 0$

Is it possible for A and B to be both **mutually exclusive** and **independent**?

a yes

b no

c there isn't enough information to decide

$B = \emptyset$

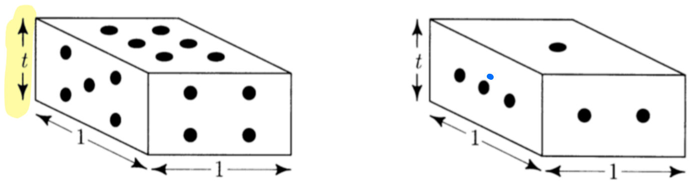
$\underbrace{P(AB)}_{=0} = P(A)\underbrace{P(B)}_{=0}$ ✓

ex

Sec 1.5 Baye's rule

P(A and B)
P(AB)
P(A,B)
P(A∩B)
all the same

Shapes.
A *shape* is a 6-sided die with faces cut as shown in the following diagram:



A box contains 3 shaped die (see pic above), D_1, D_2, D_3, with probability $\frac{1}{3}, \frac{1}{2}, \frac{2}{3}$ respectively of landing flat (with 1 or 6 on top).

Note: the numbers $\frac{1}{3}, \frac{1}{2}, \frac{2}{3}$ don't add up to 1 because they are the chance of landing flat for 3 different die.

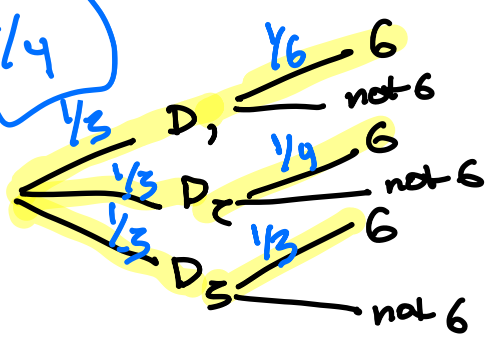
a) what is $P(\text{get } 6 \mid D_1) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

forward conditional or likelihood conditional

b) what is $P(\text{get } 6, D_1) = \underbrace{P(\text{get } 6 \mid D_1)}_{= 1/6} \cdot \underbrace{P(D_1)}_{= 1/3} = \frac{1}{18}$

c) what is $P(\text{get } 6)$

$\frac{1}{3}\cdot\frac{1}{6} + \frac{1}{3}\cdot\frac{1}{4} + \frac{1}{3}\cdot\frac{1}{3} = \boxed{\frac{1}{4}}$



d) Find posterior $P(D_1 \mid 6) = \frac{P(D_1, 6)}{P(6)} = \frac{1/18}{1/4} = \boxed{2/9}$

backwards conditional, need Baye's rule

HHHHT is <u>not</u> allowed

Suppose you draw a number from a bag, with equal probabilities across the choices $\{1, 2, 3\}$.

Once you draw a number, you toss a coin until you get exactly that many number of heads followed by a tails—so if you draw a 3, you keep tossing until you encounter the sequence {Heads, Heads, Heads, Tails}.

What is the probability of tossing a coin seven times given that you draw the number 2?

$P(7 \mid \text{draw } 2)$ — forward conditional (dont use Baye's rule)

_ _ _ T H H T

anything but

Cant be HHT

THH

$8-2=6$

$$\boxed{P(7 \mid \text{draw } 2) = \frac{6}{2^7}}$$