STAT 134: Section 10

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Conceptual Review

a. What is the moment generating function of a random variable X?

b. How do we get the k^{th} moment of X from the MGF of X?

c. If X and Y are independent, how can you write the MGF of their sum, $X+Y$, in terms of the MGFs of X and Y?

Problem 1

Let $X \sim \text{Binom}\ (n, p)$

a. Find the moment generating function of X, $M_X(t)$.

b. Use (a) to find $\mathbb{E}(X)$.

Hint: use the binomial theorem, which states that for any a, b, $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$

Problem 2

a. Recall that, if $X \sim N(0,1)$, then $f_X(x) = (2\pi)^{-1/2}e^{-x^2/2}$. Find the MGF $M_{X^2}(t)$ of X^2 by direct calculation.

Hint: substitute $u = x\sqrt{1-2t}$ and assume $t < \frac{1}{2}$.

b. Recall that the density of $Y \sim \mathrm{Exp}(\lambda)$ is $f_Y(y) = \lambda e^{-\lambda y}$ for $y \geq 0$ and zero otherwise. Again, by direct calculation, find $M_Y(t)$.

Hint: assume $t < \lambda$.

c. Conclude that if X_1 and X_2 are i.i.d. $N(0,1)$ random variables, then $X_1^2 + X_2^2 \sim \mathrm{Exp}(\frac{1}{2})$.

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