STAT 134: Section 12

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Conceptual Review

a. What is an order statistic?

b. Let X, Y have joint density $f_{X,Y}$. Draw a picture showing what we mean by $P(X \in dx, Y \in dy)$.

c. If X, Y are jointly uniformly distributed over a region, what does their joint density look like?

Problem 1

Four people agree to meet at a cafe at noon. Suppose each person arrives at a time chosen uniformly at random between 11:45 am and 12:15 pm, independently of the others.

1. What is the chance that the first person to arrive at the cafe gets there before 11:50?

2. What is the chance that some of the four have not arrived by 12:10?

3. Suppose that if all 4 people have shown up before 12:10, the waiter takes their orders as soon as the fourth person arrives. Otherwise, the waiter takes the orders of whoever is there at 12:10 pm. Let T represent the time at which the waiter takes their order. Find and sketch the cdf of T.

From Ex 4.6.1 in Pitman's Probability

Problem 2

Let $U_{(1)}, \ldots, U_{(n)}$ be the values of n i.i.d. Uniform (0,1) variables arranged in increasing order. For $0 < x < y < 1$, find simple formulae for:

a. $P(U_{(1)} > x, U_{(n)} < y)$

b. $P(U_{(1)} > x, U_{(n)} > y)$

c. $P(U_{(1)} < x, U_{(n)} < y)$

d. $P(U_{(1)} < x, U_{(n)} > y)$

Ex 4.6.3 in Pitman's Probability

Problem 3

X, Y are jointly distributed (not necessarily uniformly!) with a strictly positive density on the region $R = \{(x, y) : 1 < x < y < 3\}$ and such that $\int\int_R f_{X,Y}(x, y)dxdy = 1$. True or false: X, Y could be independent.