STAT 134: Section 14

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Conceptual Review

a. Let X, Y have joint density $f_{X,Y}(x, y) > 0$ which is strictly positive for $x, y > 0$ (and only for $x, y > 0$). Set up an integral that would yield the density of $Z = X + Y$.

b. Do the same for $Z = Y/X$.

c. For $X \sim \text{Exp}(\lambda)$, $a > 0$, what is the distribution of aX? If $X \sim \text{Exp}(\lambda)$, $Y \sim \text{Exp}(\mu)$, and X, Y are independent, what is $\mathbb{P}(X < Y)$?

d. For two discrete random variables X and Y, what do we mean by the conditional distribution of Y given $X = x$? What is an expression for the conditional expectation of Y given $X = x$?

Problem 1: Convolution of Uniforms

Let $X \sim$ Unif (0,1), and $Y \sim$ Unif (0,2), independent of each other. Find the density of $Z = X + Y$, using the convolution formula.

Problem 2: Ratio of Exponentials

Suppose X, Y are i.i.d. $\mathrm{Exp}(1)$ r.v.s. Find the density of $Z = Y/X$ using:

a. the densities of X and Y.

b. the CDF of Z.

Problem 3: A Conditional Expectation

Throw a fair, six–sided die until you get a "6." Denote by T the number of throws (including the final throw which produced a "6").

Make an educated guess: **What is the conditional expectation of T, given that all the throws resulted in even numbers?**

Now, consider the related scenario. Throw a fair, six–sided die until you get a number which is not 2 or 4.

a. What is the expected number of throws (including the final throw), S?

b. Call the result of the ith throw X_i. What is the expected value of S, given $X_S = 6$?

c. Revise your guess to the bolded question above.