STAT 134: Section 15

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Conceptual Review

a. If you know the conditional distribution of Y given X, how do you find $\mathbb{E}[Y]$?

b. For each of the following, indicate whether it is a random variable or a constant. For instance, $\mathbb{E}[X]$ is a constant.

   (a) $\mathbb{E}[Y|X]$

   (b) $\mathbb{E}[Y|X = x]$

   (c) $\mathbb{E}[\mathbb{E}[Y|X]]$

   (d) $\mathbb{E}[X|X]$

   (e) $\mathbb{E}[Y|X = Y]$

c. If you know $f_X(x)$ and $f_{X,Y}(x, y)$, how do you find $f_{Y|X}(y|x)$?

Problem 1

Let Y have exponential distribution with mean $\frac{1}{2}$. Let X be such that, conditionally on $Y = y$, X has exponential distribution with a mean of y.

a. Write $f_{X|Y}(x|y)$.

b. Write an integral which, if you computed it, would give $\mathbb{E}[X]$ (but you don't need to compute it).

c. Find $\mathbb{E}[X]$ using the properties of expectation instead.

Problem 2

Suppose that Y and Z have the following joint density:

$$f(y,z) = \begin{cases} k(z-y) & \text{for } 0 \leq y \leq z \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

for some positive constant k (its value is irrelevant). Find:

a. the marginal distribution of Y and

b. $\mathbb{P}(Z < \frac{2}{3} \mid Y = \frac{1}{2})$.