STAT 134: Section 3

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Problem 1

Given that there were 12 heads in 20 independent coin tosses, calculate the chance that

a. the first two tosses landed heads;

b. at least two of the first five tosses landed heads.

Ex 2.1.5 in Pitman's Probability

Try to do this problem with as little tedious work as possible.

Problem 2

A gambler decides to keep betting on red at roulette where there are 18 reds out of 38 tiles in total, and stop as soon as she has won a total of five bets.

a. What is the probability that she has to make exactly 8 bets before stopping?

b. What is the probability that she has to make at least 9 bets?

Ex 2.1.12 in Pitman's Probability

Problem 3: The matching problem

There are n letters addressed to n people at n different addresses. The n addresses are typed on n envelopes. A disgruntled secretary shuffles the letters and puts them in the envelopes in random order, one letter per envelope.

a. What is the chance that the i_{th} letter is put in the correctly addressed envelope? How about both i_{th} letter and j_{th} letter ($i \neq j$)? And the chance that the letters at positions $i_1, i_2, \ldots, i_k$ are put in correctly?

b. Find the probability that at least one letter is put in a correctly addressed envelope;

c. What is the probability in part b. approximately, for large n?

Hint: Use the inclusion-exclusion formula: $P(\cup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i) - \sum_{i<j} P(A_i A_j) + \ldots + (-1)^{n+1} P(A_1 A_2 A_3 \ldots A_n)$

Ex 2.rev.28 in Pitman's Probability