

STAT 134: Section 9

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Conceptual Review

Consider a Poisson Process with rate λ per unit time. What are the distributions of:

- N_t , the number of arrivals by time t ;
- W_k , the waiting time between the $(k - 1)^{\text{st}}$ and k^{th} arrivals; and
- T_k , the waiting time until the k^{th} arrival.

Problem 1

Suppose calls arrive at a telephone exchange at an average rate of one per second, according to a Poisson arrival process. Find:

- the probability that the fourth call after time $t = 0$ arrives within 2 seconds of the third call;
- the probability that the fourth call arrives by time $t = 5$ seconds;
- the expected time at which the fourth call arrives.

Ex 4.2.5 in Pitman's Probability

Problem 2: Gammas, Exponentials, and Moments

Consider the gamma function $\Gamma(r) = \int_0^\infty x^{r-1}e^{-x}dx$, $r > 0$.

- Use integration by parts to show that $\Gamma(r + 1) = r\Gamma(r)$.
- Deduce from (a) that for any positive integer n , $\Gamma(n) = (n - 1)!$
- Show that if $T \sim \text{Exp}(1)$, then $\mathbb{E}(T^n) = n!$.
- Show that if $S = T/\lambda$, then $S \sim \text{Exp}(\lambda)$. (Note: from this, we can easily show that $\mathbb{E}(S^n) = n!/\lambda^n$).

Hint: Consider the expression $P(S > s)$, then substitute for S appropriately.

Ex 4.2.9 in Pitman's Probability

Problem 3

Shocks occur to a system according to a Poisson process of rate λ . Suppose that the system survives each shock with probability α , independently of other shocks, so that its probability of surviving k shocks is α^k . What is the probability that the system is surviving at time t ?