Stat 134: Conditional Probabilities and Expectations

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Problem 1

Let X_1 and X_2 be independent and uniformly distributed on $\{1, 2, \ldots, n\}$. Let X be the minimum and Y be the maximum of X_1 and X_2. Find:

a. $E(Y|X = x)$

b. $E(X|Y = y)$

Ex 6.2.2 in Pitman's Probability

Problem 2

Suppose that N is a Poisson random variable with parameter μ. Suppose that given $N = n$, random variables $X_1, X_2, \ldots, X_n$ are independent with uniform (0,1) distribution. So there are a random number of X's.

a. Given $N = n$, what is the probability that all the X's are less than $t \in [0, 1]$

b. What is the (unconditional) probability that all the X's are less than $t \in [0, 1]$

c. Let $S_N = X_1 + ... + X_N$ denote the sum of the random number of X's. Find $P(S_N = 0)$.

d. Find $E(S_N)$

Ex 6.2.6 in Pitman's Probability

Problem 3

Let A and B be events and let Y have a uniform (0, 1) distribution. Suppose that conditioned on $Y = p$, A and B are independent with probability p. Find expressions for:

a. the conditional probability of A given that B occurs.

b. the conditional density of Y given that A occurs and B does not.

Ex 6.3.9 in Pitman's Probability

Problem 4

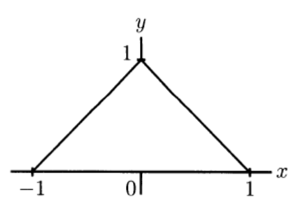
Suppose (X, Y) has uniform distribution on the triangle in the diagram. For x between -1 and 1, find:

a. $P(Y \geq \frac{1}{2} | X = x)$

b. $P(Y < \frac{1}{2} | X = x)$

c. $E(Y | X = x)$

d. $Var(Y | X = x)$



Ex 6.3.5 in Pitman's Probability