Stat 134: Review Section Worksheet: Densities + CDF + Order Statistics

Hank Ibser

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Density Practice

Suppose we have a poisson process with rate λ, i.e. $P(arrival \in dt) \approx \lambda dt$

a. What is the distribution of T_r, the time of the $r-th$ arrival, show you work. Derive directly from $P(T_r \in dt)$ using N_t.

b. As r gets very large what is the density of $(T_r - E(T_r))/SD(T_r)$? Hint: treat T_r as the appropriate sum of r.v.s.

Minimum of Exponentials

We will show that the minimum of exponentials is exponetially distributed, using multiple approaches. Let, Z = min(X, Y), where $X, Y \sim exp(\lambda)$ is exponential.

a. Show straight from the CDF, $F_Z(z)$.

b. Show directly from $P(Z \in dz)$

c. Generalize to set of n, what is the distrbution of $min(X_1, X_2, \cdots, X_n)$?

CDF Practice

Let $T \sim exp(\lambda)$, $Y = \sqrt{T}$

a. Find the density of Y, using the CDF of Y.

b. Calculate the Expectation of Y.

Modified Ex 4.5.7 in Pitman's Probability

Order Statistic Practice

Assume N points are picked independently and uniformly at random from unit circle. Let $R_1, \cdots, R_N$ denote the random radius of n points sampled in this way. Assume $N \geq 7$.

a. Find the density and CDF or R.

b. Find the density of $R_{(k)}$ for $1 \leq k \leq N$

c. Find the joint density of $R_{(3)}, R_{(7)}$

Modified from Quiz 4

Prepared by Maxwell J. Weinstein