# *Stat 134: Math Review*

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The following are exercises for approximations, summations, and alegbra. All of these techniques cannot possibly appear on one midterm exam, but any of them might.

## *Summation*

Simplify the following.

1. $1 + 2 + ... + n$
2. $2 + 4 + 6 + ... + (2n)$
3. For $m < n$, $\sum_{k=m}^{n} x^k$
4. For $|x| < 1$, $\sum_{k=0}^{\infty} x^k$
5. $\sum_{i=1}^{20} \left(\frac{1}{2}\right)^i$
6. $\sum_{i=4}^{20} \left(\frac{1}{2}\right)^i$
7. $\sum_{i=2}^{\infty} \left(\frac{1}{3}\right)^i$
8. $\sum_{i=0}^{10} \left(\frac{1}{4}\right)^i$

## *Exponential Approximation*

We can express $e^x$ as a power series:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{k=0}^{\infty} \frac{x^n}{n!}.$$

For small $x$ (meaning that $|x|$ is close to 0),

$$e^x \approx 1 + x \tag{1}$$

$$\log(1 + x) \approx x. \tag{2}$$

1. Using $\log(1 + x) \approx x$ for small $x$, show that $\left(1 - \frac{2}{n}\right)^n \approx e^{-2}$ for large $n$.
2. What is $1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + ... + (-1)^{71} \frac{1}{70!}$ approximately equal to?
3. Roll a 1000-sided die 1000 times. Find the expected number of faces that do not appear. What is this approximately equal to?

## *Basic Algebra*

Let $a$ and $b$ be real numbers and $n$ be an integer. Assume that the denominator is nonzero when division is involved. True or false:

1. T / F. $(a+b)^n = a^n + b^n$
2. T / F. $(a-b)^n = a^n - b^n$
3. T / F. $(ab)^n = a^n b^n$
4. T / F. $(a/b)^n = (a^n)/(b^n)$
5. T / F. $a^{b+c} = a^b a^c$
6. T / F. $a^{b-c} = (a^b)/(a^c)$
7. T / F. $a^{bc} = (a^b)^c$

## *Binomial Theorem*

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}.$$

1. Use binomial theorem to show that the binomial$(n, p)$ probabilities do indeed sum to 1.

## *The square of a summation*

The following identity might be useful when computing $E(Y^2)$, when $Y = X_1 + X_2 + \cdots + X_n = \sum_{i=1}^{n} X_i$:

$$\left(\sum_{i=1}^{n} a_i\right)^2 = \sum_{i=1}^{n} a_i^2 + \sum_{i \neq j} a_i a_j = \sum_{i=1}^{n} a_i^2 + 2\sum_{i<j} a_i a_j$$

1. There are $n$ letters and $n$ envelopes, and each letter has a corresponding envelope. Suppose I randomly place the $n$ letters in the $n$ envelopes (each envelope can contain only one letter). Let $X$ be the number of correctly placed letters. What is Var$(X)$? *Do not leave your answer in summation.*