# *Stat 134: Joint Distributions Review*

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## *Problem 1: Setting Boundaries*

Let random variables $X, Y$ have joint density function $f_{X,Y}(x, y)$ over the region $\{(x, y) : -1 < x < 1, 0 < y < 3\}$. For each part below, (i) set up (but do not evaluate) an integral for the given expression, and (ii) compute the exact answer in the case where the density is uniform over the region.

a. $P(X < Y)$

b. $P(X + 2Y > 1)$

c. $P(X^2 < Y)$

d. $P(\sqrt{X^2 + Y^2} > 1)$

e. $P(Y < y)$

## *Problem 2*

Consider a Poisson random scatter of points in a plane with mean intensity $\lambda$ per unit area. That is, the distribution of the number of arrivals in a region of area $s$, $N_s$, is Poisson $(s\lambda)$. Let $R$ be the distance from the origin to the closest point of the scatter.

a. Find formulae for the c.d.f. and density of $R$.

Hint: What does the event $R > r$ mean in terms of where the arrivals occurred?

b. Show that $\sqrt{2\lambda\pi}R$ has the Rayleigh distribution.

c. Use (b) to find $\mathbb{E}(R)$ and $SD(R)$.

*Ex 5.rev.21 in Pitman's Probability*

## *Problem 3*

The unit interval is cut into 3 disjoint intervals of length $p_1, p_2$, and $p_3$, where $p_1 + p_2 + p_3 = 1$. For some fixed $n$, let $U_1, U_2, \ldots, U_n$ be i.i.d. Uniform (0,1) r.v.'s. For $j = 1, 2, 3$, let $N_j$ be the number of $U$'s that fall into the region covered by $p_j$.

a. Give a formula for $P(N_1 = n_1, N_2 = n_2, N_3 = n_3)$ where $n_1, n_2, n_3 \geq 0$ and $n_1 + n_2 + n_3 = n$.

b. For $u < p_1$ and $1 \leq m,\ k < n$, find $P(U_{(k)} < u, N_1 = k + m)$.

c. Now suppose we consider $U_1, U_2, \ldots, U_N$, where $N$ is Poisson $(\lambda)$. In this case, what is the joint distribution of $(N_1, N_2, N_3)$?

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