Stat 134: Section 10

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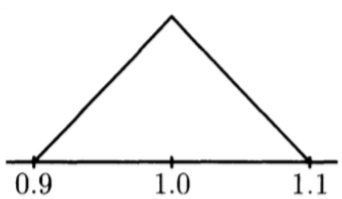
Problem 1

Suppose the distribution of height over a large population of individuals is approximately normal. Ten percent of individuals in the population are over 6 feet tall, while the average height is 5 feet 10 inches. What, approximately, is the probability that in a group of 100 people picked at random from this population there will be two or more individuals over 6 feet 2 inches tall?
Ex 4.1.7 in Pitman's Probability

Problem 2

Suppose a manufacturing process designed to produce rods of length 1 inch exactly, in fact produces rods with length distributed according to the density graphed below.



For quality control, the manufacturer scraps all rods except those with length between 0.925 and 1.075 inches before he offers them to buyers.

a. What proportion of output is scrapped?

b. A particular customer wants 100 rods with length between 0.95 and 1.05 inches. Assuming lengths of successive rods produced by the process are independent, how many rods must this customer buy to be 95% sure of getting at least 100 of the prescribed quality?

Ex 4.1.13 in Pitman's Probability

Problem 3

A piece of rock contains 10^{20} atoms of a particular substance. Each atom has an exponentially distributed lifetime with a half-life of one century. How many centuries must pass before

a. it is most likely that about 100 atoms remain;

b. there is about a 50% chance that at least one atom remains. What assumptions are you making?

Ex 4.2.2 in Pitman's Probability

Problem 4

Geometric from exponential.

a. Show that if T has exponential distribution with rate λ, then $int(T)$, the greatest integer less than or equal to T, has a geometric(p) distribution on $\{0, 1, 2, ...\}$, and find p in terms of λ.

b. Let $T_m = int(mT)/m$, the greatest multiple of $1/m$ less than or equal to T. Show that T has exponential distribution on $(0, \infty)$ for some λ, if and only if for every m there is some p_m such that mT_m has geometric(p_m) distribution on $\{0, 1, 2, ...\}$. Find p_m in terms of λ.

c. Use b) and $T_m \leq T \leq T_m + 1/m$ to calculate E(T) and SD(T), from the formulae for the mean and standard deviation of a geometric random variable.

Ex 4.2.10 in Pitman's Probability

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