Stat 134: Section 14

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Problem 1

A metal rod is l inches long. Measurements on the length of this rod are equal to $l + \epsilon$, where ϵ is random error. Assume that the errors are uniformly distributed over the range -0.1 inch to $+0.1$ inch, and are independent of each other

a. Find the chance that a measurement is less than .01 inches away from l

b. Find the chance that two measurements are less than .01 inches away from one another.

Ex 5.1.2 in Pitman's Probability

Problem 2

Suppose X and Y are independent and uniformly distributed on the unit interval $(0, 1)$. Find:

$$P(Y \geq \frac{1}{2} | Y \geq 1 - 2X)$$

Ex 5.1.3 in Pitman's Probability

Hint: draw out the regions on the unit square

Problem 3

Let $U_{(1)} \dots U_{(n)}$be the values of n independent uniform (0,1) random variables arranged in increasing order. Let $0 \leq x < y \leq 1$. Find a formula for

a. $P(U_{(1)} > x \text{ and } U_{(n)} < y)$

b. $P(U_{(1)} \leq x \text{ and } U_{(n)} < y)$

Great visualization practice:
What does this region look like in $\mathbb{R}^n$?

Ex 5.1.8 in Pitman's Probability

Problem 4

Let $X = \min(S, T)$, $Y = \max(S, T)$ for independent $S, T \sim \text{Exponential}(\lambda)$.
Let $Z = Y - X$.

a. Find the joint density of X and Y; are X and Y independent?

b. Find the joint density of X and Z; are X and Z independent?

c. Find the marginal distributions of X and Z.

Ex 5.2.9 in Pitman's Probability