Stat 134: Section 15

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Problem 1

Suppose that (X, Y) is uniformly distributed over the region $\{(x, y) : 0 < |y| < x < 1\}$. Find:

a. the joint density of (X, Y);

b. the marginal densities $f_X(x)$ and $f_Y(y)$.

c. Are X and Y are independent?

d. Find $\mathbb{E}X$ and $\mathbb{E}Y$.

A diagram is often very helpful for problems involving joint distributions of uniforms.

Ex 5.2.1 in Pitman's Probability

Problem 2

Let X and Y have joint density $f(x, y) = 90(y - x)^8$ for $0 < x < y < 1$.

a. Find $\mathbb{P}(Y > 2X)$.

b. Find the marginal density of X.

c. Fill in the blanks. The joint density f above is the joint density of the _____ and _____ of ten independent uniform $(0, 1)$ random variables.

Ex 5.2.6 in Pitman's Probability

Problem 3

Minimum and maximum of n independent exponentials. Let $X_1, X_2, \ldots, X_n$ be independent, each with exponential (λ) distribution. Let $V = \min(X_1, X_2, \ldots, X_n)$ and $W = \max(X_1, X_2, \ldots, X_n)$. Find the joint density of V and W.
Ex 5.2.10 in Pitman's Probability

Problem 4

Let X and Y be independent and normally distributed, X with mean 0 and variance 1, Y with mean 1. Suppose $\mathbb{P}(X > Y) = 1/3$. Find the standard deviation of Y.
Ex 5.3.5 in Pitman's Probability

How can you leverage your knowledge about the joint distribution of independent normal distributions to solve this problem?