Stat 134: Section 18

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Problem 1

Let X_1 and X_2 be the numbers on two independent fair-die rolls. Let X be the minimum and Y the maximum of X_1 and X_2. Calculate $\mathbb{E}[Y|X = x]$.

Ex 6.2.1 in Pitman's Probability

Problem 2

Suppose that each individual in a population produces a random number of children, and the distribution of the number of children of a single individual has mean μ. Starting with one individual, show that the expected number of descendants of that individual in the nth generation is μ^n

Ex 6.2.8 in Pitman's Probability

Hint: Condition the number of descendants in a generation on the number of descendants in the previous generation

Problem 3

A deck of cards is cut into two halves of 26 cards each. As it turns out, the top half contains 3 aces and the bottom half just one ace. The top half is shuffled, then cut into two halves of 13 cards each. One of these packs of 13 cards is shuffled into the bottom half of 26 cards, and from this pack of 39 cards, 5 cards are dealt. What is the expected number of aces among these 5 cards?

Ex 6.2.11 in Pitman's Probability

Problem 4

You flip a biased coin until you see a certain pattern in the sequence. For each of the following, find the expected number of tosses required to see the pattern.

a. Flip until you see two heads in a row ("HH")

b. Flip until you see three heads in a row ("HHH")

c. Flip until you see n heads in a row ("HHH...HHH")

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