Stat 134: Section 6

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Problem 1

A hand of five cards contains two aces and three kings. The five cards are shuffled and dealt one by one, until an ace appears.

a. Display in a table the distribution of the number of cards dealt.

b. Suppose that dealing is continued until the second ace appears. Again display the distribution of the number of cards dealt.

c. Explain why the probabilities in the second table are just those in the first in a different order.

Hint: Think about dealing off the bottom of the deck!

Ex 3.1.8 in Pitman's Probability

Problem 2

Grouping multinomial categories. Suppose that counts $(N_1, \ldots, N_m)$ are the numbers of results in m categories in n repeated trials. So $(N_1, \ldots, N_m)$ has multinomial distribution with parameters n and $p_1, \ldots, p_m$. Let $1 \leq i < j \leq m$. Answer the following questions with an explanation, but no calculation.

a. What is the distribution of N_i?

b. What is the distribution of $N_i + N_j$?

c. What is the joint distribution of $N_i,\ N_j,\ n - N_i - N_j$?

Ex 3.1.12 in Pitman's Probability

Problem 3

Let X be the number of spades in 7 cards dealt from a well-shuffled deck of 52 cards containing 13 spades. Find $\mathbb{E}X$.
Ex 3.2.6 in Pitman's Probability

Hint: Don't attempt to compute the expectation using the standard definition.

Problem 4

Suppose an airline accepted 12 reservations for a commuter plane with 10 seats. They know that 7 reservations went to regular commuters who will show up for sure. The other 5 passengers will show up with a 50% chance, independently of each other.

a. Find the probability that the flight will be overbooked, i.e., more passengers will show up than seats are available.

b. Find the probability that there will be empty seats.

c. Let X be the number of passengers turned away. Find $\mathbb{E}X$.

d. Consider the long run interpretation of the expected value. What is the greatest amount of money the airline can pay to every bumped passenger (someone who doesn't have a seat) and still make more money in the long run compared to if they sold only 10 reservations? The answer should be expressed as a multiple of the price of a ticket.

Ex 3.R.7 in Pitman's Probability