Stat 134: Conditional Probabilities, Distributions, & Expectations Review

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Problem 1

Let $X_1 \sim$ Geom (p_1), $X_2 \sim$ Geom (p_2), $X_1 \perp X_2$, both on $\{1, 2, \ldots\}$. Find:

a. $P(X_1 \leq X_2)$;

b. $P(X_1 = x \mid X_1 \leq X_2)$. Recognize $X_1 | X_1 \leq X_2$ as a named distribution, and state the parameter(s).

Problem 2

Let $Y \sim$ Beta (r, s). Conditioned on $Y = y$, let $X \sim$ Geometric (y) on $\{0, 1, 2, \ldots\}$ For simplicity, assume $r, s > 1$.

a. What is $\mathbb{E}(X \mid Y = y)$?

b. Find $\mathbb{E}(X)$.

c. Find $P(X = x)$, for $x \in \{0, 1, 2, \ldots\}$.

Problem 3

Suppose a proportion p of a population has a gene m that makes them predisposed to migraines. Of these people, the number of migraines they experience in a year follows a Poisson process with rate μ per year, whereas the rest of the population experiences migraines according to a Poisson process with rate λ.

a. What is the probability that a randomly selected individual experiences no migraines in a given year?

Hint: Condition on whether the individual has gene m.

b. Let N_t denote the number of migraines a randomly selected individual experiences in t years. Find $\mathbb{E}(N_t)$.

c. Find $Var(N_t)$.

Problem 4

Let X, Y have joint density $f_{X,Y}(x, y) = 2\lambda^2 e^{-\lambda(x+y)}, 0 < x < y$. It can be shown that $f_X(x) = 2\lambda e^{-2\lambda x}, x > 0$. Find:

a. The conditional density of Y, given $X = x$;

b. $\mathbb{E}(Y|X = x)$.

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