Stat 134 lec 7

Warmup 10:00–10:10

p large

large

ex 97.8% of approx 30 million poor families in the U.S. have a fridge. If you randomly sample 100 of these families roughly what is the chance 98 or more have a fridge?

Defn Poisson(μ)

$$P(k) = \frac{e^{-\mu}\mu^k}{k!} \quad \text{for } k=0,1,2,\dots$$

p = prob have a fridge = .978

$n = 100$

$\mu = (.978)(100)$

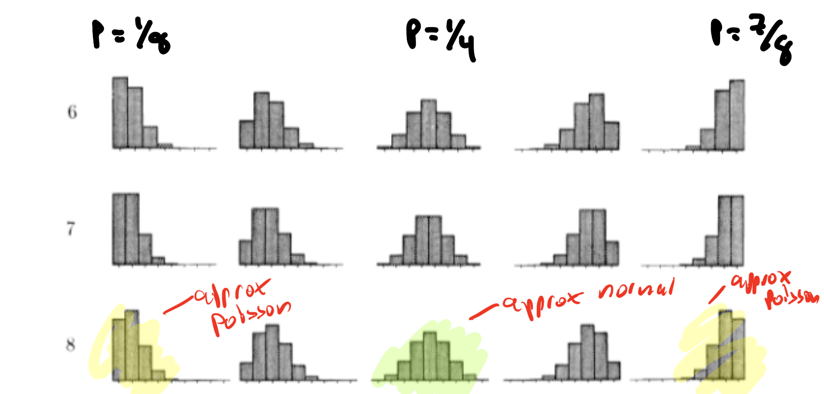
P(98 or more out of 100 have fridge)

= P(2 or less don't have a fridge)

$= P(0) + P(1) + P(2)$

use Pois($\mu = nq$) = Pois(2.2)

$$\approx e^{-2.2} + \frac{e^{-2.2}(2.2)^1}{1!} + \frac{e^{-2.2}(2.2)^2}{2!}$$



Last time

sec 2.4 **Poisson Distribution**

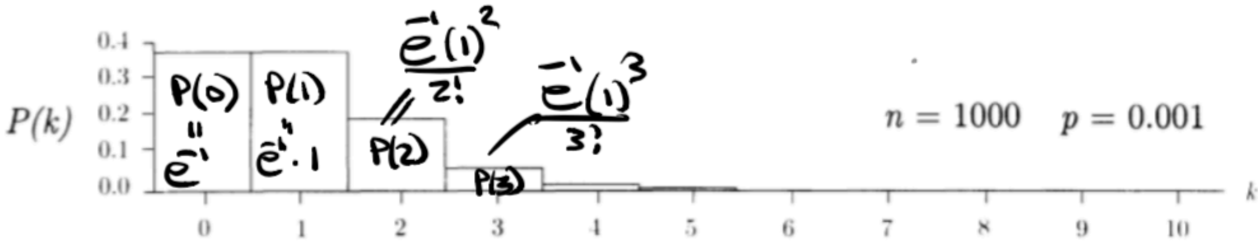
$$P(k) = \frac{e^{-\mu}\mu^{k}}{k!}, \quad k = 0, 1, 2, \ldots$$

We saw that Pois(μ) is a limit of binomials for $n \to \infty$, $p \to 0$ and $np \to \mu$

or $q \to 0$ $nq \to \mu$.

The binomial (1000, 1/1000) distribution.

Now take 1000 random draws with replacement from a box with 1 black ball and 999 white ones. This is the distribution of the number of black balls drawn:



mode of Bin(n,p):

$m = \lfloor np+p \rfloor$

$$\text{mode} = \begin{cases} m & \text{if } np+p \notin \mathbb{Z} \\ m-1, m & \text{if } np+p \in \mathbb{Z} \end{cases}$$

mode of Pois(μ):

$m = \lfloor \mu \rfloor$ since $np+p \to \mu+p \approx \mu$

$$\text{mode} = \begin{cases} m & \text{if } \mu \notin \mathbb{Z} \\ m-1, m & \text{if } \mu \in \mathbb{Z} \end{cases}$$

Today

(1) **sec 2.5** Random sampling

independent trials (draw w/ replacement)
- binomial distribution — 2 outcome trial
- multinomial distribution — k outcome trial

dependent trials (draw w/o replacement)
- hypergeometric distribution — 2 outcome trial
- multivariate hypergeometric distribution — k outcome trial

1 Sec 2.5

Random sampling with replacement

ex Class 100 students

grade distribution:

A 50 students

B 30 students

C 15 students

D 5 students

You sample 10 students with replacement.

a) What is the chance you get

AAAABBBCCD ? AAABBBCCDA

$(.5)^4(.3)^3(.15)^2(.05)^1$

b) Find $P(4\ A's, 3\ B's, 2\ C's, 1\ D)$

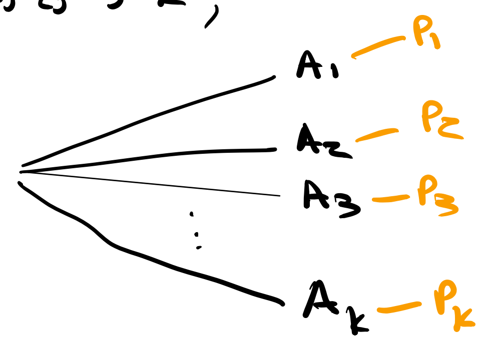
$\binom{10}{4,3,2,1}\cdot(.5)^4(.3)^3(.15)^2(.05)^1$

$\frac{10!}{4!\,3!\,2!\,1!} = \binom{10}{4,3,2,1}$

$= \binom{10}{4}\cdot\binom{6}{3}\binom{3}{2}\binom{1}{1}$

Def^{n} Multinomial Distribution written $Multi(n, p_1, \dots, p_k)$

If you have n independent trials, where each trial has k possible outcomes, $A_1, A_2, \dots, A_k$ with probabilities $p_1, p_2, \dots, p_k$,



then the probability you get n_1 outcome A_1, n_2 outcome A_2, ..., n_k outcome A_k is

$$P(n_1, n_2, \dots, n_k) = \binom{n}{n_1, n_2, \dots, n_k} p_1^{n_1} p_2^{n_2} \cdots p_k^{n_k}$$

$$\frac{n!}{n_1! \, n_2! \cdots n_k!}$$

Note Binomial distribution is a special case with $k=2$.

independent trials (draw w/ replacement) $\begin{cases} \text{binomial distribution — 2 outcome trial} \\ \text{multinomial distribution — } k \text{ outcome trial} \end{cases}$

random sample without replacement

ex In a very student friendly class with 100 students the grade distribution is:

A 70 students
B 30 students

You sample 5 students at random without replacement (called a simple random sample (SRS))

a) Find the chance you get

A A A B B

$$\frac{70}{100}\cdot\frac{69}{99}\cdot\frac{68}{98}\cdot\frac{30}{97}\cdot\frac{29}{96} = \frac{70}{100}\cdot\frac{69}{99}\frac{30}{98}\frac{29}{97}\cdot\frac{68}{96}$$

A A B B A

b) Find P(3A's, 2B's).

A A B B A

5 4 3 2 1

$$\frac{5!}{3!\,2!}\cdot\frac{70}{100}\frac{69}{99}\frac{68}{98}\cdot\frac{30}{97}\cdot\frac{29}{96} = \frac{\frac{70\cdot69\cdot68}{3!}\cdot\frac{30\cdot29}{2!}}{\frac{100\cdot99\cdot98\cdot97\cdot96}{5!}}$$

$$\frac{5!}{3!\,2!} = \binom{5}{3,2}$$

$$= \frac{\binom{70}{3}\binom{30}{2}}{\binom{100}{5}}$$

hyper-geometric formula

written

Def^n hypergeometric distribution $HG(n, N, G)$

Suppose a population of size N contains G good and B bad elements ($N = G + B$).
A sample, size n, with g good and b bad elements ($n = g + b$) is chosen at random without replacement.

$$P(g \text{ good and } b \text{ bad}) = \frac{\binom{G}{g}\binom{B}{b}}{\binom{N}{n}}$$

this generalizes to the **multivariate hypergeometric distribution**

Now instead of 2 types of elements we have K with sizes $G_1, G_2, \ldots, G_K$ ($N = G_1 + \cdots + G_K$) and in our sample we have $n = g_1 + \cdots + g_K$.

$$P(g_1, g_2, \ldots, g_K) = \frac{\binom{G_1}{g_1}\binom{G_2}{g_2}\cdots\binom{G_K}{g_K}}{\binom{N}{n}}$$

ex Class 100 students
grade distribution:

A	50	students
B	30	students
C	15	students
D	5	students

You sample 10 students without replacement (SRS)

Find $P(4 A's, 3 B's, 2 C's, 1 D) = \dfrac{\binom{50}{4}\binom{30}{3}\binom{15}{2}\binom{5}{1}}{\binom{100}{10}}$

ex A 5 card <u>poker hand</u> consists of a SRS of 5 cards from a 52 card deck. There are $\binom{52}{5}$ poker hands.

a) Find P(poker hand has 4 aces and a king)

$$\frac{\overset{\text{aces}}{\binom{4}{4}}\overset{\text{kings}}{\binom{4}{1}}}{\binom{52}{5}}\binom{44}{0} \qquad \binom{44}{0} = 1$$

n

b) Find P(poker hand has 4 aces).

$$\frac{\overset{\text{aces}}{\binom{4}{4}}\overset{\text{non aces}}{\binom{48}{1}}}{\binom{52}{5}} = \frac{\binom{12}{1}\binom{4}{4}\binom{4}{1}}{\binom{52}{5}}$$

c) Find P(poker hand has 4 of a kind)

pick 4 of a kind

pick 1 of a kind given your choice of the 4 of a kind

$$\frac{\binom{13}{1}\binom{4}{4}\binom{12}{1}\binom{4}{1}}{\binom{52}{5}}$$

For next time think about

P(a poker hand has two 2 of a kind)

ex King, King, Queen, Queen, 7