Stat 134 lec 16

Warmup

A tub of blueberry muffin batter has $\lambda = 2\ ^{bb}/_{in^3}$ intensity of bb.

bb muffin 1 is $3\ in^3$

bb muffin 2 is $4\ in^3$

a) on average how many bb is in muffin 1?

$\mu_1 = \lambda \cdot 3 = \boxed{6 \text{ bb}}$

b) Find P(5 bb in each muffin)

$X_1 \sim Pois(6)$

$X_2 \sim Pois(8)$

$P(X_1 = 5, X_2 = 5) = \boxed{\frac{e^{-6} 6^5}{5!} \cdot \frac{e^{-8} 8^5}{5!}}$

c) Find P(10 bb total in both muffins together).

$X_1 + X_2 \sim Pois(14)$

$P(X_1 + X_2 = 10) = \boxed{\frac{e^{-14} 14^{10}}{10!}}$

Announcements

For Monday's review, write down questions in discussion board on b-course by Sunday 8pm.

Last time

Sec 3.5 Poisson distribution

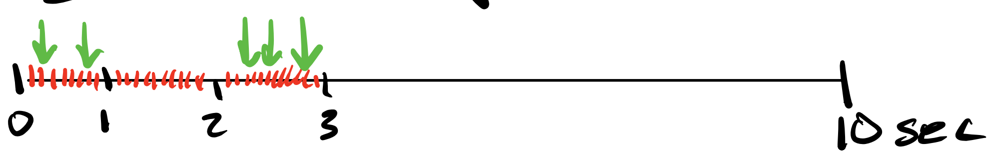
$$X \sim Pois(\mu)$$

$$P(X=k) = \frac{e^{-\mu}\mu^{k}}{k!}$$

$$E(X) = Var(X) = \mu.$$

Poisson Process or Poisson Random Scatter (PRS):

ex radioactive decay of Americium 241 in 10 seconds



Assumptions

① no two particles arrive at the same time. (this allows us to divide 10 sec into n small time intervals each with at most one arrival.

② X is a sum of n ← large iid Bernoulli(p) trials.
↖ small

$\mu = np$ is avg # of arrivals in 10 sec.
$\lambda = \mu/10$ is the arrival rate per second.

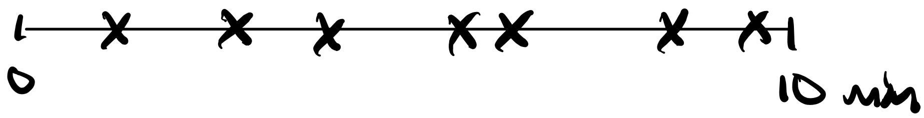
X = # arrivals in 10 seconds.
Suppose $\lambda = 4$ arrivals/sec
then $\mu = \lambda \cdot 10 = 40 \Rightarrow X \sim \text{Pois}(40)$
Americium has a long half life.
Y = # arrivals in 12070 sec.
$Y \sim \text{Pois}(\lambda \cdot 12070)$
4

Today ① Finish section 3.5 Poisson Thinning
② midterm review

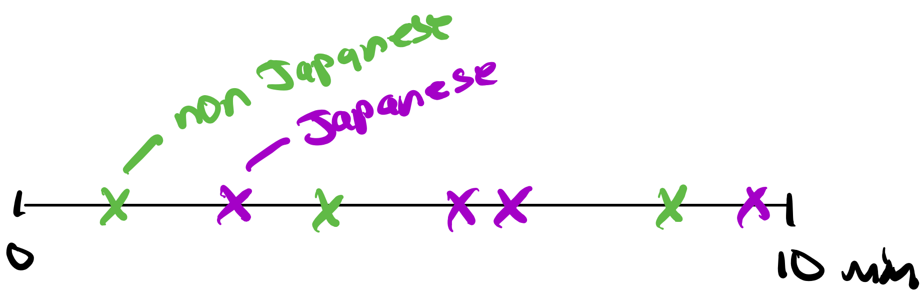
Sec 3.5 Poisson thinning

Cars arrive at a toll booth according to a Poisson process at a rate $\lambda = 3$ arrivals/min

X = # cars arriving at a toll booth in 10 min. $X \sim Pois(\underbrace{\lambda \cdot 10}_{30})$



Of cars arriving, it is known, over the long term, that 60% are Japanese imports.



Call Japanese cars a success and non Japanese a failure.

The arrival of Japanese cars is is a a "thinned" Poisson Process of the arrival of all types of cars.

cars $\sim$ Pois$(\lambda \cdot 10)$ = Pois(30)

Japanese imports $\sim$ Pois$(p\lambda \cdot 10)$ = Pois(18)

non Japanese $\sim$ Pois$(q\lambda \cdot 10)$ = Pois(12)

Ex What is the probability that in a given 10 min interval, 15 cars arrive at the booth and 10 are Japanese imports?

$$P(X=15, J=10) = P(nJ=5, J=10)$$

$$= P(nJ=5)P(J=10)$$

$$= \boxed{\frac{e^{-12}\,12^{5}}{5!} \cdot \frac{e^{-18}\,18^{10}}{10!}}$$

② midterm review

Which distributions are exactly or (approximately) a sum of a fixed number of independent Bernoulli trials?

Discrete

name and range	$P(k) = P(X = k)$ for $k \in$ range	mean	variance
uniform on $\{a, a+1, \ldots, b\}$ $\{1,2,\ldots,n\}$	$\frac{1}{b-a+1}$	$\frac{a+b}{2}$ $\frac{n+1}{2}$	$\frac{(b-a+1)^2-1}{12}$ $\frac{n^2-1}{12}$
Bernoulli (p) on $\{0,1\}$ ✓	$P(1) = p$; $P(0) = 1-p$	p	$p(1-p)$
binomial (n,p) on $\{0,1,\ldots,n\}$ ✓	$\binom{n}{k} p^k (1-p)^{n-k}$	np	$np(1-p)$
Poisson (μ) on $\{0,1,2,\ldots\}$ ✓	$\frac{e^{-\mu}\mu^k}{k!}$	μ	μ
✗ hypergeometric (n,N,G) on $\{0,\ldots,n\}$	$\frac{\binom{G}{k}\binom{N-G}{n-k}}{\binom{N}{n}}$	$\frac{nG}{N}$	$n\left(\frac{G}{N}\right)\left(\frac{N-G}{N}\right)\left(\frac{N-n}{N-1}\right)$
✗ geometric (p) on $\{1,2,3\ldots\}$	$(1-p)^{k-1}p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
✗ geometric (p) on $\{0,1,2\ldots\}$	$(1-p)^k p$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$
✗ negative binomial (r,p) on $\{0,1,2,\ldots\}$	$\binom{k+r-1}{r-1} p^r (1-p)^k$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$
✓ normal	$\Phi(x)$	μ	σ^2

sum of dependent indicators

$X = I_1 + I_2 + \cdots + I_n$

$\frac{G}{N}$

$I_2 = \begin{cases} 1 & \text{if } 2^{nd} \text{ card good} \\ 0 & \text{else} \end{cases}$

CLT ⇒ normal is a sum of iid Bernoulli (p) trials, DeMorgan's rule: $(A\cap B)^c = A^c \cup B^c$

$\Rightarrow A\cap B = (A^c \cup B^c)^c$

so $\boxed{P(A\cap B) = 1 - P(A^c \cup B^c)}$

Inclusion exclusion formula:

Let A_1, A_2, A_3 be dependent RVs with $P(A_i) = .9$ for $i = 1,2,3$.

Find a lower bound for $P\left(\bigcap_{i=1}^{3} A_i\right)$

$$P\left(\bigcap_{i=1}^{3} A_i\right) = 1 - P\left(\bigcup_{i=1}^{3} A_i^c\right)$$ DeMorgan's rule,

$$P\left(\bigcup_{i=1}^{3} A_i^c\right) = P(A_1^c) + P(A_2^c) + P(A_3^c) - P(A_1^c A_2^c) - \quad - \quad + P(A_1^c A_2^c A_3^c)$$

$$\le \underbrace{P(A_1^c) + P(A_2^c) + P(A_3^c)}_{= 3(.1) = .3}$$

$$\Rightarrow P\left(\bigcap_{i=1}^{3} A_i\right) \ge 1 - .3 = \boxed{.7}$$

expectation question

An urn contains 90 marbles, of which there are 20 greens, 20 blacks and 50 red marbles. Tom draw marble without replacement until the 6^{th} green marble. Let X = # of marbles drawn. Example: **GGG**BRB**GG**BR**G** with $x = 11$. Find $\mathbb{E}[X]$.

<u>Hint</u> First find the expected number of marbles until the 1^{st} green marble.

What is the min and max of X?

X = # marbles until first green. — 1 – 71

$p = 1/21$

$X = I_1 + \dots + I_{70} + 1$ ← green marble at the end.

$E(X) = 70\left(\frac{1}{21}\right) + 1$

$I_2 = \begin{cases} 1 & \text{if } 2^{nd} \text{ nG before } 1^{st} \text{ green} \\ 0 & \text{else.} \end{cases}$

$1 — G_2 — G_3 — G_6 — G_{20} —$

↑ nG here.

Y = # marbles until 6^{th} green

$$\boxed{E(Y) = 6\left(70\left(\frac{1}{21}\right) + 1\right)}$$

↑ by symmetry,

ex Conditional distribution, Poisson

8. Let X_1 and X_2 be independent random variables such that for $i = 1, 2$, the distribution of X_i is Poisson (μ_i). Let m be a fixed positive integer. Find the distribution of X_1 given that $X_1 + X_2 = m$. Recognize this distribution as one of the famous ones, and provide its name and parameters.

$$P(X_1 = k) = \frac{e^{-\mu_1} \mu_1^{k}}{k!}$$