Stat 134 lec 4

Warmup:

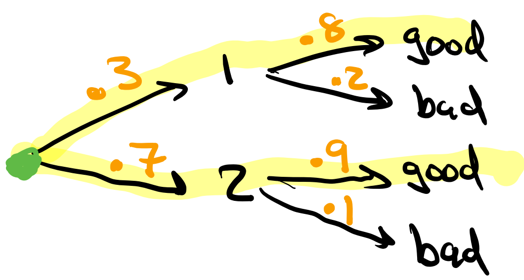
A factory produces 2 models of cell phones.

Given $P(1) = .3$ ← prior prob

$P(\text{good} \mid 1) = .8$

$P(\text{good} \mid 2) = .9$

forward conditional



Find $P(1 \mid \text{good})$

backwards conditional

$$= \frac{P(1, \text{good})}{P(\text{good})} \qquad P(1,\text{good}) = (.3)(.8) = .24$$

$$P(\text{good}) = (.3)(.8) + (.7)(.9) = .87$$

$$= \boxed{.28}$$

Last time

If A and B are indep then so is A, B^c, and A^c, B and A^c, B^c.

sec 1.5 **Bayes' rule**

There are two types of conditional probabilities:

ex

$P(\text{good} \mid 1)$ is forward conditional (likelihood conditional)

DON'T NEED BAYES TO COMPUTE

$P(1 \mid \text{good})$ is backwards conditional (posterior conditional)

NEED BAYES TO COMPUTE

Suppose A and B are two events with

$$P(A) = 0.8 \text{ and } P(A \cup B) = 0.8.$$

Is it possible for A and B to be both **mutually exclusive** and **independent**?

a yes

b no

c there isn't enough information to decide

b	Mutually exclusive implies dependency

true if two events are nonempty.

a	P(B) = 0 makes A and B ME and independent

Today

① sec 1.6 independence of 3 or more events

② sec 2.1 Binomial Distribution

sec 1.6 Independence of 3 events

Def^n (pairwise independence of 3 events)

A, B, C are pairwise independent if

$P(AB) = P(A)P(B)$ and $P(AC) = P(A)P(C)$ and $P(BC) = P(B)P(C)$

ex

> One ball is drawn randomly from a bowl containing four balls numbered 1, 2, 3, and 4. Define the following three events:
>
> - Let A be the event that a 1 or 2 is drawn. That is, $A = \{1, 2\}$.
> - Let B be the event that a 1 or 3 is drawn. That is, $B = \{1, 3\}$.
> - Let C be the event that a 1 or 4 is drawn. That is, $C = \{1, 4\}$.

Is A, B, C pairwise independent?

$\underbrace{P(AB)}_{= 1/4} = \underbrace{P(A)}_{= 1/2}\underbrace{P(B)}_{= 1/2}$ ✓

Similarly $\underbrace{P(AC)}_{= 1/4} = P(A)P(C)$, $\underbrace{P(BC)}_{= 1/4} = P(B)P(C)$ ✓

Is $\underbrace{P(ABC)}_{= 1/4} \neq \underbrace{P(A)}_{= 1/2}\underbrace{P(B)}_{= 1/2}\underbrace{P(C)}_{= 1/2}$

Def^n (mutual independence of 3 events)

A, B, C are mutually independent if

$P(ABC) = P(A)P(B)P(C)$, (and the same for any of the events replaced by its complement)

$P(A^c BC) = P(A^c)P(B)P(C)$

$P(A^c B^c C)$

$P(ABC)$

We require showing 8 equations is true for mutual independence. This is a strong condition.

<u>Thm</u> Suppose A, B, C are mutually independent. Then they are also pairwise independent.

Pf/ We can write

$$P(AB) = \underbrace{P(ABC) + P(ABC^c)}_{\text{add}^n \text{ rule}}$$

$$= P(A)P(B)P(C) + P(A)P(B)P(C^c)$$

$$= P(A)P(B)\left[P(C) + P(C^c)\right]$$

$$= P(A)P(B).$$

Similar for other cases $P(AC) = P(A)P(C)$ etc.

$\square$

Note that $P(ABC) = P(A)P(B)P(C)$ by itself doesn't imply pairwise independence:

ex let $\Omega = \{1,2,3,4,5,6,7,8\}$

$A = B = \{1,2,3,4\}$ $\qquad ABC = \{1\}$

$C = \{1,5,6,7\}$

Is $\underbrace{P(ABC)}_{= 1/8} = \underbrace{P(A)}_{= 1/2}\underbrace{P(B)}_{= 1/2}\underbrace{P(C)}_{= 1/2}$?

Is A, B, C pairwise indep?

$\underbrace{P(AB)}_{= 1/2} \neq \underbrace{P(A)}_{= 1/2}\underbrace{P(B)}_{= 1/2}$

Thm A, B, C are mutually independent iff
1) A, B, C are pairwise indep
2) $P(ABC) = P(A)P(B)P(C)$.

Sketch

Suppose (1) and (2) hold,

let's show $P(ABC^c) = P(A)P(B)P(C^c)$

write

$$P(ABC^c) = \underbrace{P(ABC^c) + P(ABC)}_{= P(AB) = P(A)P(B)} - \underbrace{P(ABC)}_{= P(A)P(B)P(C)}$$

$$= P(A)P(B)\left[1 - P(C)\right]$$

$$= P(A)P(B)P(C^c)$$

similar for other cases. $\square$

(2) Sec 2.1 Binomial distribution.

Bernoulli(p) trial (distribution)

two outcomes — success — p
— failure — 1−p

ex roll a die.
success is getting a 6 — 1/6
failure is not getting a 6 — 5/6

Binomial (n, p) distribution (Bin(n, p))

we have n independent Bernoulli(p) trials

n: fixed

p: fixed (unconditional probability)

ex we roll a die n times.

What are the possible number of successes?
0, 1, 2, .., n

In Bin(n, p) the chance of having k successes ($0 \le k \le n$) is given by the Binomial formula:

$$P(k) = \frac{n!}{k!\,(n-k)!}\, p^{k}\,(1-p)^{n-k}$$

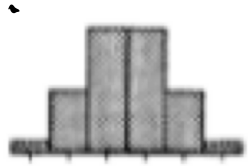
n: # trials

p: chance of success.

k: number of successes

ex You roll a die 5 times. What is the chance of getting 2 sixes?

$n = ?$ — 5
$k = ?$ — 2
$p = ?$ — $\frac{1}{6}$



$$P(2) = \frac{5!}{2!\,3!}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = \frac{5\cdot 4\cdot \cancel{3\cdot 2\cdot 1}}{2\cdot 1\cdot \cancel{3\cdot 2\cdot 1}}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3$$

$$= \boxed{10\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3}$$

What is chance of getting

success (6)

failure (not 6)

How many of these are there?

$1\ 1\ 0\ 0\ 0$? — $\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3$

$0\ 1\ 1\ 0\ 0$?

$\vdots$

$$\binom{5}{2} = \frac{5!}{2!\,3!}$$

We write $\frac{5!}{2!\,3!}$ as $\binom{5}{2}$ or $\binom{5}{3}$ or $\binom{5}{2,3}$

$\binom{5}{2} = \frac{5!}{2!\,3!}$, $\binom{5}{3} = \frac{5!}{3!\,2!}$

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ex

1. Ten cards are dealt off the top of a well shuffled deck. The binominal formula doesn't apply to find the chance of getting exactly three diamonds because:

   a The probability of a trial being successful changes

   b The trials aren't independent

   c There isn't a fixed number of trials

   d more than one of the above

A trial is whether a card in the deck is a diamond or not. There are 10 trials.

The trials are dependent but each trial has the same unconditional probability (1/4) of being a diamond.