Stat 134 lec 7

Warmup!

Suppose you and I each have a box of 600 marbles. In my box, 4 of the marbles are black, while 3 of your marbles are black. We each draw 300 marbles **with replacement** from our own boxes. **Approximately**, what is the chance you and I draw the same number of black marbles?

Def$^n$ Poisson$(\mu)$

$$P(k) = \frac{e^{-\mu}\mu^k}{k!} \text{ for } k=0,1,2,\dots$$

Hint Using a Poisson Approx to the Binomial what is the chance you get k blacks?

$\mu_Y = 300\left(\frac{3}{600}\right) = 1.5$

$$\frac{e^{-1.5}(1.5)^k}{k!}$$

X = # black marbles (out of 300) I draw

Y = # black you draw

$$P(X=Y) = \sum_{k=0}^{300} P(X=k, Y=k) \quad \text{addition rule}$$

$$= \sum_{k=0}^{300} P(X=k)P(Y=k)$$

$$= \boxed{\sum_{k=0}^{300} \frac{e^{-2}2^k}{k!} \cdot \frac{e^{-1.5}1.5^k}{k!}}$$

Last time

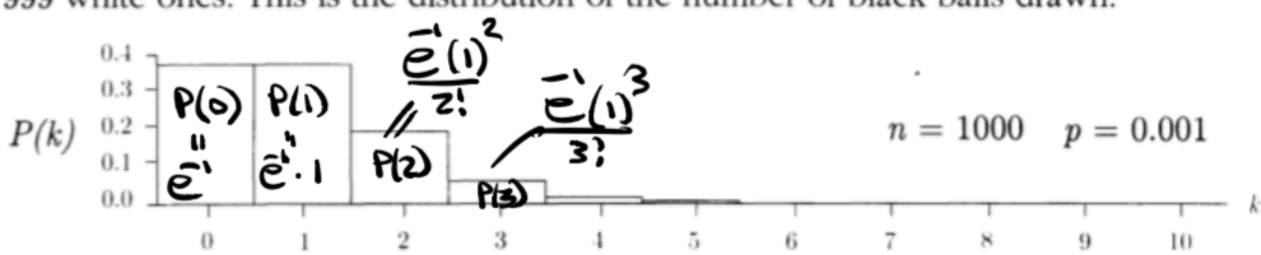
sec 2.4 Poisson Distribution

$$P(k) = \frac{e^{-\mu}\mu^{k}}{k!}, \quad k = 0, 1, 2, \ldots$$

We saw that Pois($\mu$) is a limit of binomials for $n \to \infty$, $p \to 0$ and $np \to \mu$

**The binomial (1000, 1/1000) distribution.**

Now take 1000 random draws with replacement from a box with 1 black ball and 999 white ones. This is the distribution of the number of black balls drawn:



stat 134 concept test

Southwest overbooks its flights, knowing some people will not show up. Suppose for a plane that seats 350 people, 360 tickets are sold. Based on previous data the airline claims that each passenger has a 90% chance of showing up. **Approximately**, what is the chance that at least one empty seat remains? (There are no assigned seats)

a) $P(Z < \frac{350.5-\mu}{\sigma})$

b) $P(Z < \frac{349.5-\mu}{\sigma})$

c) $P(Z < \frac{360.5-\mu}{\sigma})$

d) none of the above

1 − chance no empty seats

350, 351, 352, . . .

$1 - P(Z > \frac{349.5-\mu}{\sigma})$

$= P(Z \leq \frac{349.5-\mu}{\sigma})$

| | |
|---|---|
| d | we ~~need~~ to apply the compliment rule |
| b | we want the area to the left of 350 (not including 350 itself, since we want at least one open seat). the formula in (b) thus gives us P(X < 350). |

Today

(1) sec 2.5 Random sampling

independent trials (draw w/ replacement)
- binomial distribution — 2 outcome trial
- multinomial distribution — k outcome trial

dependent trials (draw w/o replacement)
- hypergeometric distribution — 2 outcome trial
- multivariate hypergeometric distribution — k outcome trial

① Sec 2.5

Random sampling with replacement

ex Class 100 students

grade distribution:

| | | |
|---|---|---|
| A | 50 | students |
| B | 30 | students |
| C | 15 | students |
| D | 5 | students |

You sample 10 students with replacement.

a) What is the chance you get

AAAABBBCCD ? $(.5)^4(.3)^3(.15)^2(.05)^1$

AABAABBBCCD same answer

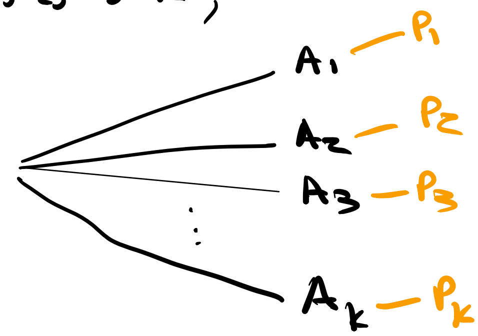
b) Find P(4 A's, 3 B's, 2 C's, 1 D)

$$\binom{10}{4,3,2,1}\cdot(.5)^4(.3)^3(.15)^2(.05)^1$$

$$= \frac{10!}{4!3!2!1!} = \binom{10}{4}\cdot\binom{6}{3}\binom{3}{2}\cdot\binom{1}{1}$$

**Def^n** <u>Multinomial Distribution</u> written $\text{Multi}(n, p_1, \dots, p_k)$

If you have $n$ independent trials, where each trial has $k$ possible outcomes, $A_1, A_2, \dots, A_k$ with probabilities $p_1, p_2, \dots, p_k$,




then the probability you get $n_1$ outcome $A_1$, $n_2$ outcome $A_2$, ..., $n_k$ outcome $A_k$ is

$$P(n_1, n_2, \dots, n_k) = \binom{n}{n_1, n_2, \dots, n_k} p_1^{n_1} p_2^{n_2} \cdots p_k^{n_k}$$

$$\frac{n!}{n_1! \, n_2! \cdots n_k!}$$

<u>Note</u> Binomial distribution is a special case with $k=2$.

independent trials (draw w/ replacement) $\left\{ \begin{array}{l} \text{binomial distribution — 2 outcome trial} \\ \text{multinomial distribution — } k \text{ outcome trial} \end{array} \right.$

## random sample without replacement

ex In a very student friendly class with 100 students the grade distribution is:

A 70 students
B 30 students

You sample 5 students at random without replacement (called a simple random sample (SRS))

a) Find the chance you get

A A A B B

$$\frac{70}{100}\ \frac{69}{99}\ \frac{68}{98}\ \frac{30}{97}\ \frac{29}{96}$$

b) Find P(3A's, 2B's).

$$\binom{5}{3,2}\frac{70}{100}\ \frac{69}{99}\ \frac{68}{98}\ \frac{30}{97}\ \frac{29}{96} = \frac{\frac{70\cdot 69\cdot 68}{3!}\cdot\frac{30\cdot 29}{2!}}{\frac{100\cdot 99\cdot 98\cdot 97\cdot 96}{5!}}$$

$$\binom{5}{3,2} = \frac{5!}{3!\,2!}$$

$$= \frac{\overset{A}{\binom{70}{3}}\ \overset{B}{\binom{30}{2}}}{\binom{100}{5}}$$

hypergeometric formula

Def$^n$ hypergeometric distribution

written
$HG(n, N, G)$

Suppose a population of size $N$ contains $G$ good and $B$ bad elements ($N = G + B$).
A sample, size $n$, with $g$ good and $b$ bad elements ($n = g + b$) is chosen at random without replacement.

$$P(g \text{ good and } b \text{ bad}) = \frac{\binom{G}{g}\binom{B}{b}}{\binom{N}{n}}$$

this generalizes to the multivariate hypergeometric distribution

Now instead of 2 types of elements we have $K$ with sizes $G_1, G_2, \dots, G_K$ ($N = G_1 + \dots + G_K$) and in our sample we have $n = g_1 + \dots + g_K$.

$$P(g_1, g_2, \dots, g_K) = \frac{\binom{G_1}{g_1}\binom{G_2}{g_2}\cdots\binom{G_K}{g_K}}{\binom{N}{n}}$$

ex Class 100 students
grade distribution:

| | |
|---|---|
| A | 50 students |
| B | 30 students |
| C | 15 students |
| D | 5 students |

You sample 10 students without replacement (SRS)

Find $P(4A\text{'s}, 3B\text{'s}, 2C\text{'s}, 1D) = \dfrac{\binom{50}{4}\binom{30}{3}\binom{15}{2}\binom{5}{1}}{\binom{100}{10}}$

ex A 5 card poker hand consists of a SRS of 5 cards from a 52 card deck. There are $\binom{52}{5}$ poker hands.

a) Find P(poker hand has 4 aces and a king)

$$\frac{\overset{\text{Aces}}{\binom{4}{4}}\overset{\text{Kings}}{\binom{4}{1}}}{\binom{52}{5}}$$

choose your single (say king)

n

b) Find P(poker hand has 4 aces)

Ace    king

$P(AB)=P(A)P(B|A)$

$$\frac{\binom{4}{4}\binom{48}{1}}{\binom{52}{5}} = \frac{\binom{12}{1}\binom{4}{4}\binom{4}{1}}{\binom{52}{5}}$$

c) Find P(poker hand has 4 of a kind)

quad    single    Ace    king

$$\frac{\binom{13}{1}\binom{12}{1}\binom{4}{4}\binom{4}{1}}{\binom{52}{5}}$$

aaaab    $a \neq b$

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**Stat 134**

1. The probability of being dealt a three of a kind poker hand (ranks $\underbrace{aaabc}_{aaac}$ where $a \neq b \neq c$) is:

   **a** $\binom{4}{3}\binom{4}{1}\binom{44}{1}/\binom{52}{5}$

   rank of triple

   **b** $\binom{13}{1}\underline{\binom{12}{2}}\binom{4}{3}\binom{4}{1}\binom{4}{1}/\binom{52}{5}$

   **c** $\binom{13}{1}\binom{12}{1}\binom{4}{3}\binom{4}{1}\binom{44}{1}/\binom{52}{5}$

   ''

   $\binom{11}{1}\binom{4}{1}$

   **d** none of the above

Notice that aaabc = aaacb in your poker hand so we have $\binom{12}{2}$ in numerator not $\binom{12}{1}\binom{11}{1}$

Also note that a correct answer would also be $\binom{13}{2}\binom{11}{1}\binom{4}{3}\binom{4}{1}\binom{4}{1}/\binom{52}{5}$

since $\binom{13}{2}\binom{11}{1} = \frac{13 \cdot 12}{2} \cdot \frac{11}{1}$

and $\binom{13}{1}\binom{12}{2} = \frac{13}{1} \cdot \frac{12 \cdot 11}{2}$

equal.

ex What is probability you have two, 2 of a kind in your poker hand aabbc? $a \neq b \neq c$.

Answer: doubles singles

$$\frac{\binom{13}{2}\binom{11}{1}\binom{4}{2}\binom{4}{2}\binom{4}{1}}{\binom{52}{5}} = \frac{\binom{13}{1}\binom{12}{2}\binom{4}{2}\binom{4}{2}\binom{4}{1}}{\binom{52}{5}}$$